

TU DORTMUND

MASTER THESIS

# **Analysis of Load Profiles in the SimBench Dataset**

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## List of Abbreviations

- BDEW: Bundesverband der Energie- und Wasserwirtschaft (German Association of Energy and Water Industries)
- CCR: Correct Classification Rate
- DTW: Dynamic Time Warping
- H0: Household Standard Load Profile
- L1: Agricultural Standard Load Profile type 1
- L2: Agricultural Standard Load Profile type 2
- LOESS: Locally Estimated Scatterplot Smoothing
- LV: Low-Voltage
- MANOVA: Multivariate Analysis of Variance
- ANOVA: Analysis of Variance
- MSE: Mean Squared Error
- p.u.: Per Unit
- SLP: Standard Load Profile
- SSCP: Sum of Squares and Cross-Products

## List of Notations

- $n$ : Node index,  $n \in \{1, \dots, N\}$
- $N$ : Total number of nodes ( $N = 5$ )
- $d$ : Day index,  $d \in \{1, \dots, D\}$
- $D$ : Total number of days ( $D = 366$ )
- $h$ : Quarter-hour interval index,  $h \in \{1, \dots, H\}$
- $H$ : Total number of quarter-hour intervals per day ( $H = 96$ )
- $T$ : Total observations per node ( $T = D \cdot H = 35,136$ )
- $t$ : Global time index,  $t \in \{1, \dots, T\}$
- $P_{n,d,h}$ : Active power at node  $n$ , day  $d$ , interval  $h$  (in p.u.)
- $y_{n,t}$ : Active power at node  $n$  at global time step  $t$
- $\tilde{y}_{n,t}$ : Stage 1 smoothed active power at node  $n$ , time  $t$
- $\tilde{P}_{n,d,h}^{(1)}$ : Stage 1 smoothed active power at node  $n$ , day  $d$ , interval  $h$
- $\tilde{P}_{n,d,h}$ : Final (Stage 2) smoothed active power at node  $n$ , day  $d$ , interval  $h$
- $\tilde{P}_{n,d}$ : Empirical load profile vector at node  $n$ , day  $d$  ( $\in \mathbb{R}^{96}$ )
- $w_1$ : Half-window size for Stage 1 smoothing
- $w_2$ : Window size for Stage 2 smoothing
- $W_t$ : Centered local neighborhood of time step  $t$
- $\alpha$ : Smoothing parameter for exponential smoothing,  $\alpha \in (0, 1)$
- $\alpha_{LOESS}$ : Span parameter for LOESS, controlling proportion of observations used in local fit
- $\alpha_{sig}$ : Significance level for hypothesis testing
- $\delta(t_0)$ : Local neighborhood radius at query point  $t_0$  in LOESS
- $u_\tau(t_0)$ : Tricube kernel weight assigned to observation  $\tau$  at query point  $t_0$
- $w$ : Weekday type index,  $w \in \{1, \dots, 7\}$
- $M_w$ : Number of non-holiday occurrences of weekday  $w$  in the year ( $M_w \approx 52$ )
- $z_{i,h}^{n,w}$ : Stage 1 smoothed active power at node  $n$ , interval  $h$ ,  $i$ -th occurrence of weekday  $w$
- $\mathcal{H}$ : Set of public holidays
- $k$ : Number of clusters ( $k = 3$ )

- $\mu_\ell$ : Centroid of cluster  $C_\ell$
- $d_{DTW}(x, y)$ : Dynamic Time Warping distance between profiles  $x$  and  $y$
- $C(p, q)$ : Cumulative cost matrix element in DTW
- $d_{corr}(x, y)$ : Correlation-based distance between profiles  $x$  and  $y$
- $s(m)$ : Silhouette value for the  $m$ -th profile
- $a(m)$ : Mean intra-cluster distance for the  $m$ -th profile
- $b(m)$ : Mean distance to nearest neighbouring cluster for the  $m$ -th profile
- $\bar{s}$ : Average Silhouette score
- $\hat{\pi}$ : Optimal cluster-to-season label assignment (Hungarian algorithm)
- CCR: Correct Classification Rate
- $\mathbf{Z}_{r,s,\delta}$ : Response vector of the  $r$ -th observation in group  $(s, \delta)$ ,  $\in \mathbb{R}^{96}$
- $s$ : Season level: 0 = winter, 1 = transition, 2 = summer
- $\delta$ : Day type level: 1 = weekday, 2 = Saturday, 3 = Sunday
- $\boldsymbol{\mu}$ : Overall population mean vector
- $\boldsymbol{\alpha}_s$ : Main effect of season  $s$  in MANOVA model
- $\boldsymbol{\beta}_\delta$ : Main effect of day type  $\delta$  in MANOVA model
- $\boldsymbol{\gamma}_{s,\delta}$ : Season-by-day-type interaction effect in MANOVA model
- $\boldsymbol{\epsilon}_{r,s,\delta}$ : Error vector,  $\sim \mathcal{N}_{96}(\mathbf{0}, \boldsymbol{\Sigma})$
- $\mathbf{E}$ : Error SSCP matrix
- $\mathbf{H}$ : Hypothesis SSCP matrix
- $V^{(s)}$ : Pillai's trace statistic
- $\nu_E$ : Error degrees of freedom
- $\nu_H$ : Hypothesis degrees of freedom
- $p$ : Dimension of response vector ( $p = 96$ )
- $R_{r,s,\delta}$ : Residual profile vector (empirical profile minus SLP)
- $\text{SLP}_{s,\delta}$ : Normalized BDEW Standard Load Profile for season  $s$ , day type  $\delta$

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# 1. Introduction

Power distribution grids supply electricity to a variety of consumer types. To model the expected electricity demand of these consumers, the German Association of Energy and Water Industries (BDEW) provides standardized daily consumption patterns. These patterns are known as standard load profiles (SLPs). The SLPs describe the theoretical power consumption behavior by consumer type, season, and day type. A challenge is that theoretical expectations of consumption often do not align with observed data at the node level. Yet it remains unclear whether these standards apply to the specific active power time series in the SimBench dataset. This thesis aims to assess whether the active power time series at five specific nodes match the corresponding BDEW SLPs.

To answer this question, three statistical methods are used. First, smoothing techniques are applied to reduce noise in the daily active power time series. Second, clustering is performed using three distance measures. This step aims to determine whether the groups match the season and day type categories defined in the BDEW SLPs. Lastly, Multivariate Analysis of Variance (MANOVA) tests if season and day type have a significant effect on the active power time series, and whether these effects remain after subtracting the SLPs from the data. If the data truly follows the SLPs, the differences caused by season and day type should mostly disappear after this step.

The SimBench rural low-voltage grid dataset (version 1, scenario 0, no-switch variant) from 2016 is used in this study. It contains quarter-hourly active power measurements at 13 nodes representing household (H0) and agricultural (L1, L2) consumer types. Five representative nodes are selected based on prior correlation-based clustering (Agarwal, 2026). These are Nodes 1 and 12 as agricultural representatives, and Nodes 8, 11, and 13 as household representatives. This study only focuses on active power (P).

The results show that the BDEW SLPs consistently overestimate active power use at all five nodes. Seasonal and day type effects remain significant in the residuals. Only Node 12 shows some visual similarity to its SLP shape, but clustering and MANOVA still suggest a significant difference in behavior. This indicates that the actual power data does not follow the SLP structure at each node.

The thesis is conducted as follows. Section 2 describes the research question, the SimBench dataset, the grid layout, and the BDEW SLP framework. Section 3 explains the statistical methods, including the four smoothing models, K-means clustering, evaluation metrics, and the MANOVA design. Section 4 presents the results in three parts. First, it shows the smoothed empirical load profiles and compares them visually with the SLPs. Second, it presents the clustering results for each node and distance metric. Third, it reports the MANOVA results for both tests. Section 5 provides the conclusion and the answers

to this study questions. Section 6 discusses the research limitations and suggests future research directions.

## 2. Problem statement

This section establishes the study objective and describes the data foundation for the analysis. This section has four parts. First, Section 2.1 outlines the research question. Next, Section 2.2 describes the SimBench dataset. Third, Section 2.3 presents the structure of the power distribution grid. Finally, Section 2.4 introduces the BDEW SLPs.

### 2.1. Research question

The question of this thesis is how closely the active power time series recorded at selected nodes in the SimBench rural low-voltage grid follow the structure prescribed by the BDEW SLPs. It can be split into three statistical questions:

- Do the empirical load profiles (the smoothed daily active power time series) share the seasonal and day type patterns shown in the SLPs?
- Do clustering algorithms applied to these daily active power time series recover groupings that correspond to the season and day type categories defined by the SLP framework?
- Do season and day type have a statistically significant effect on the daily active power time series, and does this effect become smaller when the corresponding SLPs are subtracted from the data?

These questions are addressed separately for each of the five representative nodes.

### 2.2. SimBench dataset

This study uses load profile data from the `1-LV-rural1-0-no_sw` Simbench dataset in 2016. SimBench was a research project that was conducted between 2015 and 2019. It was conducted by the University of Kassel, Fraunhofer IEE, RWTH Aachen University, and TU Dortmund, with advisory support from six German distribution network operators (SimBench Consortium, 2020, p. 1). The dataset was developed by the University of Kassel and Fraunhofer IEE. The dataset provides synthetic load profiles derived from real measurement data for German low-voltage distribution networks. The SimBench code `1-LV-rural1-0-no_sw` consists of six components separated by hyphens (SimBench Consortium, 2020, p. 87-89):



- 1: the SimBench version
- LV: low-voltage levels
- **rural11**: the urbanization character of the grid
- the empty fourth component refers to the subnet of the lower voltage level grid
- 0: scenario 0, this scenario denotes today’s grid conditions without voltage or current violations
- **no\_sw**: voltage levels occur in the variant with an incomplete switch (only open line and transformer switches).

The dataset contains 13 nodes. Based on the correlation matrix results (Agarwal, 2026), five representative nodes are selected for the analysis. These nodes represent two consumer types present in the network. Nodes 8, 11, and 13 are household nodes assigned to the H0 profile, while Nodes 1 and 12 are agricultural nodes assigned to the L1 and L2 profiles, respectively. The dataset contains 35,136 observations per node. It represents 96 quarter-hourly intervals per day over 366 days in 2016. The LV grid is rated at 0.4 kV and is supplied by a single 160 kVA transformer (SimBench Consortium, 2020, p. 70). The network in this dataset follows a radial topology. This structure is characterised by its simplicity and low infrastructure cost. A more detailed description of the distribution grid structure is provided in Section 2.3.

Each node contains the following variables:

- **time**: Timestamp corresponding to each 15-minute measurement interval
- **P (active power)**: Per-unit (p.u.) scaling factor representing the temporal shape of active power consumption (SimBench Consortium, 2020, p. 94)
- **profile**: Load profile classification assigned to each node (H0 for household nodes, L1 and L2 for agricultural nodes)

The following additional variables are derived to support the analysis:

- **weekday\_name**: Name of the day of the week
- **season**: Season category (summer, transition, or winter) based on the BDEW SLP seasonal definitions
- **day\_type**: Day type category (Weekdays, Saturday, or Sunday)

No missing values or data quality issues are identified in the dataset, as the SimBench load profiles are generated with full temporal consistency across the entire year.

### 2.3. Power distribution grid

Figure 1 illustrates the 1-LV-rural1 network topology from the SimBench dataset. It represents a standardized LV distribution grid in a rural region. The distribution grid is structured as a radial distribution system. A defining characteristic of this topology is that power flows strictly in one direction from the substation toward the end nodes with no meshing or alternative supply routes between branches (Kersting, 2001, p. 908-912).

As in Figure 1, the power is distributed from Node 4 through several downstream feeders. The network consists of 13 nodes. Node 4 serves as the slack node. It supplies all remaining load profiles. From Node 4, the network branches into four main feeders and reaches the remaining 12 consumer nodes at varying depths. Some feeders are short, such as the direct connection to Node 1, while others extend across multiple intermediate nodes. The longest feeder extends from Node 4 through Nodes 7, 12, 14 and 6 to Node 5. Consumer nodes are classified into two types. The household nodes (Nodes 8, 11, and 13) are shown as squares. The remaining consumer nodes are agricultural nodes and are shown as circles. This aligns with the H0 and L1/L2 profile categories defined in the BDEW classification system.

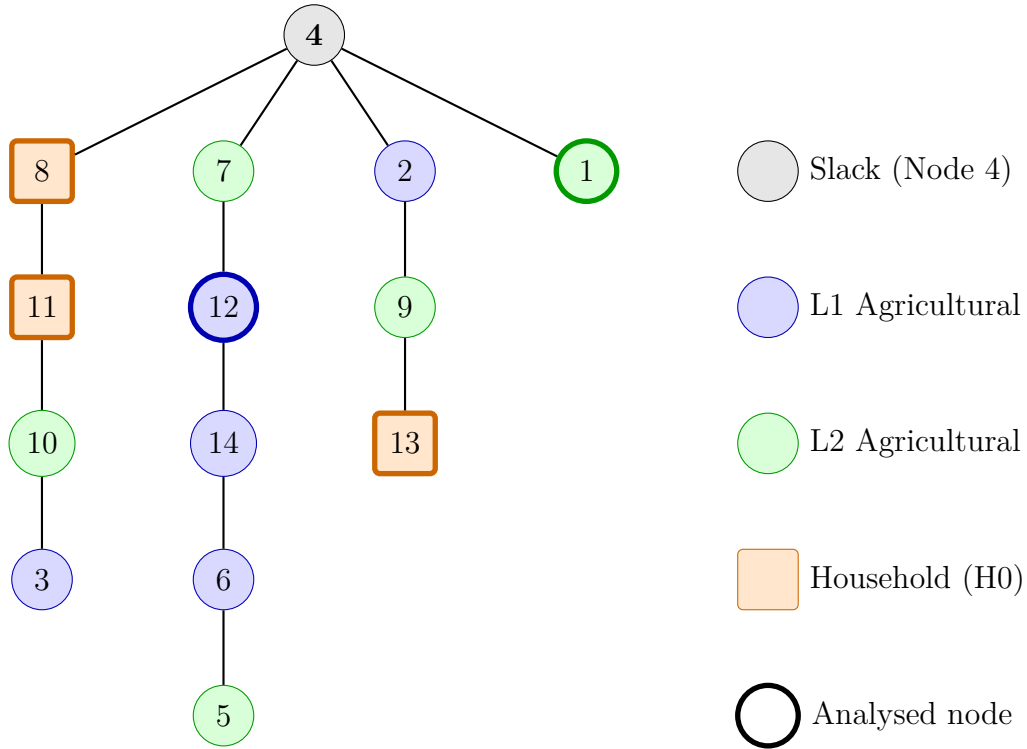


Figure 1: Power distribution grid network topology. Bold borders indicate the five nodes selected for analysis (Nodes 1, 8, 11, 12, 13).

## 2.4. Standard Load Profile (SLP)

The BDEW SLP is used for this study. The BDEW SLP provides reference low-voltage (LV) electricity consumption behavior in German distribution networks. These profiles are derived from measurements across different consumer categories. They provide normalized daily load curves that reflect usage patterns throughout the year. The BDEW defines 11 profile types across three consumer groups. They are household, commercial, and agricultural.

The SLP dataset defines (BDEW Bundesverband der Energie- und Wasserwirtschaft e.V., 2023, p. 3-5):

### Seasonal periods

- Summer: May 15 to September 14
- Winter: November 1 to March 20
- Transition season: March 21 to May 14 and September 15 to October 31

### Day types

- Weekdays: Monday to Friday
- Saturdays: December 24 and 31 are considered a Saturday unless they fall on a Sunday
- Sundays: Sundays and all public holidays

**Public holidays:** The following nine public holidays observed nationwide across all German states are considered as Sundays by default

- New Year's Day: January 1
- Good Friday
- Easter Monday
- Labour Day: May 1
- Ascension Day
- Whit Monday
- German Unity Day: October 3
- Christmas Day: December 25
- Boxing Day (2nd Christmas Day): December 26

This study focuses on household (H0) and agricultural (L1, L2) profiles. These profiles correspond to the five representative load profiles discussed in Section 2.2. Figure 2 displays three SLP types across three day types and three seasons in 2016. Seasonal patterns

are represented by orange for summer, green for transition, and purple for winter. Each subplot distinguishes between weekdays, Saturdays and Sundays. Each SLP curve represents a single theoretical daily pattern defined by the BDEW for each season and day type combination. SLPs are originally defined in watts per 1,000 kWh annual consumption (BDEW Bundesverband der Energie- und Wasserwirtschaft e.V., 2023, p. 3). For comparability with the SimBench empirical load profiles, the SLP values are normalized by dividing each profile's annual maximum value by the profile type.

The household H0 profile has a gradual morning rise and an evening peak on weekdays and Saturdays, with the morning peak appearing later on Sundays. The agricultural profiles L1 and L2 exhibit daily peaks in the morning and evening. They remain consistent across all day types. However, L2 shows a shallower midday trough between the two peaks compared to L1. Across all three profile types, winter profiles consistently show the highest consumption levels, followed by transition and summer.

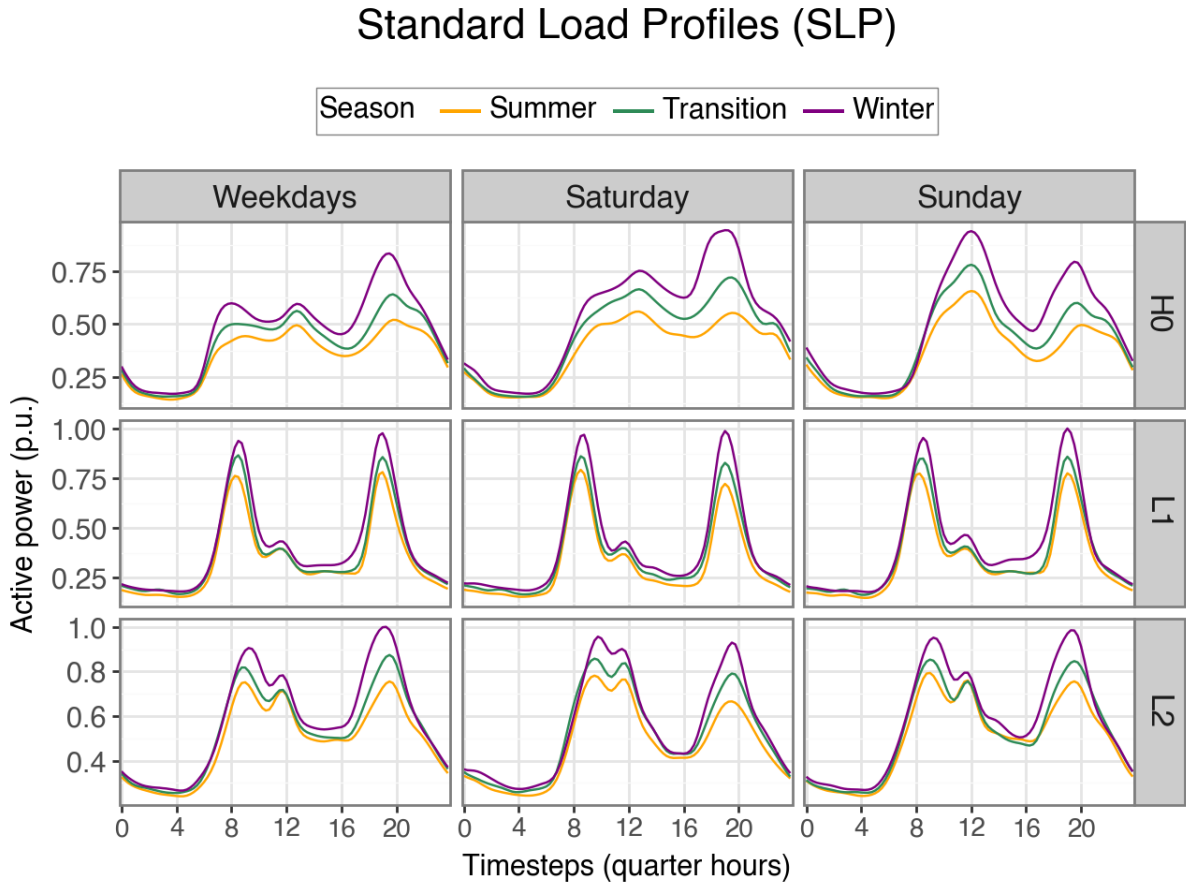


Figure 2: Standard Load Profile in 2016 (p.u.)

### 3. Statistical methods

This section describes the statistical methods used to analyze the five representative active power time series. The methods a structured pipeline designed to extract and statistically test seasonal and day type patterns in the empirical load profiles. First, Section 3.1 presents the four smoothing models applied to the empirical load profiles. Second, Section 3.2 describes the clustering models used to identify seasonal patterns. Lastly, Section 3.3 introduces two-way Multivariate Analysis of Variance (MANOVA) used to test the effects of season and day type.

The programming language, Python (Python Core Team, 2026, Version 3.13.2), and statistical software is used for analysis. All calculations and visualizations are conducted with packages as follows: NumPy (Harris and et al., 2020, Version 2.4.4), pandas (pandas development team, 2020, Version 2.3.0), matplotlib (Hunter, 2007, Version 3.10.8), seaborn (Waskom, 2021, Version 0.13.2), plotnine (Kibirige et al., 2022, Version 0.16.0), scipy (scipy Consortium Virtanen et al., 2020, Version 1.15.3), statsmodels (Seabold and Perktold, 2010, Version 0.14.6), scikit-learn (Pedregosa et al., 2011, Version 1.8.0), tslearn (Duckworth, 2012, Version 0.8.0 tslearn, pykalman (Novikov, 2019, and pyclustering

#### 3.1. Smoothing models

Smoothing is applied to the active power time series at each node to obtain empirical load profiles representative of typical consumption patterns. Thus, the procedure follows a two-stage structure.

The general notation is introduced here and used consistently throughout all four smoothing methods. Let  $n \in \{1, \dots, N\}$  denote the node index, where  $N = 5$  nodes are studied. Let  $d \in \{1, \dots, D\}$  denote the day index, with  $D = 366$  days for the leap year 2016. Let  $h \in \{1, \dots, H\}$  denote the quarter-hour interval index, where the  $H = 96$  intervals cover each 24-hour day. Thus, the total number of observations per node is  $T = D \cdot H = 35,136$ . The active power observed at node  $n$ , day  $d$ , and interval  $h$  is denoted  $P_{n,d,h}$  (in units).

For Stage 1, the observation  $P_{n,d,h}$  is re-indexed by a single global time index  $t = (d - 1)H + h \in \{1, \dots, T\}$ , so that  $t$  counts continuously from the first 15-minute interval of the year to the last. This yields the annual time series (Müller, 2025, p. 2)

$$y_{n,t} := P_{n,d,h}, \quad t = (d - 1)H + h, \quad n = 1, \dots, N, \quad (1)$$

where  $y_{n,t}$  denotes the active power at node  $n$  at global time step  $t$ .

Each smoothing method is applied to the full annual series  $\{y_{n,t}\}_{t=1}^T$  for each node  $n$  separately. The smoothed power series of Stage 1 is denoted  $\tilde{y}_{n,t}$ . After Stage 1, the

smoothed series is mapped back to the three-index form via  $\tilde{P}_{n,d,h}^{(1)} := \tilde{y}_{n,(d-1)H+h}$ , where  $\tilde{P}_{n,d,h}^{(1)}$  denotes the Stage 1 smoothed active power at node  $n$ , day  $d$ , and interval  $h$ . The superscript (1) indicates that this is the result of Stage 1 only.

After Stage 1, days that fall on public holidays are removed from the dataset before Stage 2 is applied. Let  $\mathcal{H}$  denote the set of public holidays in the study year 2016. The smoothed series restricted to non-holiday days is used as input to Stage 2.

For Stage 2, let  $w(d) \in \{1, \dots, 7\}$  denote the weekday of day  $d$ , where 1 corresponds to Monday and 7 to Sunday. For each weekday type  $w \in \{1, \dots, 7\}$ , the ordered sequence of all non-holiday days of that weekday type is collected as (Müller, 2025, p. 2-3)

$$\{d_1^w < d_2^w < \dots < d_{M_w}^w\} := \{d \in \{1, \dots, D\} : w(d) = w, d \notin \mathcal{H}\}, \quad (2)$$

where  $d_i^w$  denotes the  $i$ -th non-holiday occurrence of weekday  $w$  in the year, and  $M_w$  denotes the total number of such occurrences ordered chronologically, with  $M_w \approx 52$ . For example, if  $w = 1$  (Monday), then  $d_1^1$  is the first non-holiday Monday of the year,  $d_2^1$  is the second, and so on up to  $d_{M_1}^1$ . For each node  $n$ , weekday type  $w$ , and interval  $h$ , the cross-week sequence is defined as (Müller, 2025, p. 2-3)

$$z_{i,h}^{n,w} := \tilde{P}_{n,d_i^w,h}^{(1)}, \quad i = 1, \dots, M_w, \quad (3)$$

where  $z_{i,h}^{n,w}$  denotes the Stage 1 smoothed active power at node  $n$  and interval  $h$  on the  $i$ -th non-holiday occurrence of weekday  $w$ . That is,  $z_{1,h}^{n,w}$  is the value at the first occurrence of that weekday,  $z_{2,h}^{n,w}$  at the second occurrence, and so forth across the  $M_w \approx 52$  occurrences throughout the year. Stage 2 smoothing is applied to the sequence  $\{z_{i,h}^{n,w}\}_{i=1}^{M_w}$  along the occurrence index  $i$  independently for each combination of node  $n$ , weekday type  $w$ , and interval  $h$ . The smoothed value from Stage 2 is denoted  $\tilde{P}_{n,d,h}$ , where the absence of the superscript (1) indicates the final smoothed result. The resulting empirical load profile for node  $n$  on day  $d$  is the  $H$ -dimensional vector

$$\tilde{P}_{n,d} = \left( \tilde{P}_{n,d,1}, \dots, \tilde{P}_{n,d,H} \right)^\top, \quad (4)$$

where  $\tilde{P}_{n,d}$  collects the smoothed active power values at all  $H = 96$  quarter-hour intervals of day  $d$  for node  $n$  into a single profile vector.

### 3.1.1. Moving Averages

The moving average is a classical non-parametric smoothing technique that replaces each observation with the arithmetic mean of a symmetric local window of neighboring obser-

uations (Brockwell and Davis, 2002, p. 25). Its simplicity and computational efficiency make it widely used in time series analysis.

### 3.1.1.1. Stage 1 smoothing

The centered moving average is applied to the annual time series  $\{y_{n,t}\}_{t=1}^T$ . For each node  $n$ , the Stage 1 smoothed value at time  $t$  is defined as (Kapucugil-İkiz, 2014, p. 19)

$$\tilde{y}_{n,t} = \frac{1}{|W_t|} \sum_{\tau \in W_t} y_{n,\tau}, \quad t = 1, \dots, T, \quad (5)$$

where  $\tilde{y}_{n,t}$  denotes the smoothed active power of Stage 1 at node  $n$  and time  $t$ , and  $W_t$  is the centered local neighborhood of time step  $t$ . It is defined as

$$W_t = \{\tau \in \{1, \dots, T\} : |\tau - t| \leq w_1\}, \quad (6)$$

where  $w_1 \in \mathbb{N}$  is the half-window size, so that  $W_t$  contains all time steps within  $w_1$  steps of  $t$ . The quantity  $|W_t|$  denotes the number of elements in  $W_t$ . At interior time points,  $|W_t| = 2w_1 + 1$ ; at the boundaries of the series, the window is truncated and only the available observations are used. The resulting smoothed series  $\tilde{y}_{n,t}$  is subsequently mapped back to the three-index form,  $\tilde{P}_{n,d,h}^{(1)}$ , the Stage 1 smoothed active power at node  $n$ , day  $d$ , and interval  $h$ .

### 3.1.1.2. Stage 2 smoothing

For each node  $n$ , weekday type  $w$ , and interval  $h$ , the centered moving average is applied to the cross-week sequence  $\{z_{i,h}^{n,w}\}_{i=1}^{M_w}$ . The smoothed value of Stage 2 at occurrence  $i$  is (Kapucugil-İkiz, 2014, p. 19)

$$\tilde{P}_{n,d_i^w,h} = \frac{1}{|W_i|} \sum_{j \in W_i} z_{j,h}^{n,w}, \quad i = 1, \dots, M_w, \quad (7)$$

where  $\tilde{P}_{n,d_i^w,h}$  denotes the final smoothed active power at node  $n$ , interval  $h$ , on the day corresponding to the  $i$ -th occurrence of weekday  $w$ , and  $W_i$  is the centered neighborhood over occurrence indices defined as

$$W_i = \{j \in \{1, \dots, M_w\} : |j - i| \leq \lfloor \frac{w_2}{2} \rfloor\}, \quad (8)$$

where  $w_2 \in \mathbb{N}$  is the window size in number of occurrences and  $\lfloor \cdot \rfloor$  denotes the floor function. The quantity  $|W_i|$  is the number of occurrences within the window, which is at most  $w_2$  but may be smaller near the boundaries of the sequence. Repeating this procedure for all  $h = 1, \dots, H$  yields the empirical load profile  $\tilde{P}_{n,d_i^w} = (\tilde{P}_{n,d_i^w,1}, \dots, \tilde{P}_{n,d_i^w,H})^\top$ .

### 3.1.2. Moving Medians

The moving median, also known as the running median, is a robust alternative to the moving average. This smoothing model replaces each observation with the median of its local neighborhood (Tukey, 1977, p. 19). The median is substantially less sensitive to extreme values and outliers than the mean.

#### 3.1.2.1. Stage 1 smoothing

For each node  $n$ , the centered moving median is applied to the annual series  $\{y_{n,t}\}_{t=1}^T$  at node  $n$  and global time step  $t$ . The Stage 1 smoothed value at time  $t$  is defined as (Tukey, 1977, p. 210-211)

$$\tilde{y}_{n,t} = \text{median}\{y_{n,\tau} : \tau \in W_t\}, \quad t = 1, \dots, T, \quad (9)$$

where  $\tilde{y}_{n,t}$  denotes the Stage 1 smoothed active power at node  $n$  and time  $t$ , and  $W_t$  is the centered local neighborhood. Then, the smoothed series  $\tilde{y}_{n,t}$  is subsequently mapped back to the smoothed active power of Stage 1  $\tilde{P}_{n,d,h}^{(1)}$  at node  $n$ , day  $d$ , and interval  $h$ .

#### 3.1.2.2. Stage 2 smoothing

For each node  $n$ , weekday type  $w$ , and interval  $h$ , the centered moving median is applied to the cross-week sequence  $\{z_{i,h}^{n,w}\}_{i=1}^{M_w}$ . The Stage 2 smoothed value at occurrence  $i$  is (Tukey, 1977, p. 210-211)

$$\tilde{P}_{n,d_i^w,h} = \text{median}\{z_{j,h}^{n,w} : j \in W_i\}, \quad i = 1, \dots, M_w, \quad (10)$$

where  $\tilde{P}_{n,d_i^w,h}$  denotes the final smoothed active power at node  $n$ , interval  $h$ , on the day corresponding to the  $i$ -th occurrence of weekday  $w$ , and  $W_i$  is the centered neighborhood over occurrence indices as defined in (8). Repeating this procedure for all  $h = 1, \dots, H$  yields the empirical load profile  $\tilde{P}_{n,d_i^w} = (\tilde{P}_{n,d_i^w,1}, \dots, \tilde{P}_{n,d_i^w,H})^\top$ .

### 3.1.3. Exponential smoothing

Simple Exponential smoothing is a recursive smoothing technique in which recent observations receive larger weights than past observations, with weights decreasing exponentially over time (Brown, 1956, p. 2-3). Unlike the moving average and moving median, Exponential smoothing continuously adapts to changes in the local level of the series.

#### 3.1.3.1. Stage 1 smoothing

For each node  $n$ , the annual series  $\{y_{n,t}\}_{t=1}^T$  is smoothed recursively. The Exponential



smoothing of Stage 1 is defined as (Hyndman, 2014, p. 34)

$$\tilde{y}_{n,t} = \alpha y_{n,t-1} + (1 - \alpha) \tilde{y}_{n,t-1}, \quad t = 2, \dots, T, \quad (11)$$

with initial value  $\tilde{y}_{n,1} = y_{n,1}$ , where  $\tilde{y}_{n,t}$  denotes the Stage 1 smoothed active power at node  $n$  and time  $t$ .  $\alpha$  is the smoothing parameter that controls how much weight is placed on the most recent observation  $y_{n,t-1}$  relative to the previous smoothed value  $\tilde{y}_{n,t-1}$ . The smoothed value  $\tilde{y}_{n,t}$  can equivalently be expressed as a geometrically weighted sum of all past observations (Hyndman, 2014, p. 35)

$$\tilde{y}_{n,t} = \alpha \sum_{\ell=0}^{t-2} (1 - \alpha)^\ell y_{n,t-1-\ell} + (1 - \alpha)^{t-1} y_{n,1}, \quad (12)$$

where the weight  $(1 - \alpha)^\ell$  assigned to the observation  $\ell$  steps in the past decreases exponentially with  $\ell$ . Larger values of  $\alpha$  give more weight to recent observations and result in less smoothing. On the other hand, smaller values produce heavier smoothing. The smoothing parameter  $\alpha$  is selected by minimizing the mean squared error between the observed and smoothed values over the annual series (Hyndman, 2014, p. 35)

$$\hat{\alpha} = \arg \min_{\alpha \in (0,1)} \text{MSE}(\alpha) = \arg \min_{\alpha \in (0,1)} \frac{1}{T} \sum_{t=1}^T (y_{n,t} - \tilde{y}_{n,t})^2, \quad (13)$$

where  $\text{MSE}(\alpha)$  denotes the mean squared error as a function of  $\alpha$  and  $\hat{\alpha}$  is the optimal smoothing parameter. The relationship between the Exponential smoothing parameter and an equivalent moving average window of size  $w$  can be approximated as  $\alpha \approx 2/(w+1)$ . The smoothed series  $\tilde{y}_{n,t}$  is then subsequently mapped back to the three-index form. Thus, the Stage 1 smoothed active power  $\tilde{P}_{n,d,h}^{(1)}$  at node  $n$ , day  $d$ , and interval  $h$  is produced.

### 3.1.3.2. Stage 2 smoothing

Exponential smoothing is applied to the cross-week sequence  $\{z_{i,h}^{n,w}\}_{i=1}^{M_w}$  for each node  $n$ , weekday type  $w$ . The Stage 2 smoothed value at occurrence  $i$  is (Hyndman, 2014, p. 34)

$$\tilde{P}_{n,d_i^w,h} = \alpha_2 z_{i,h}^{n,w} + (1 - \alpha_2) \tilde{P}_{n,d_{i-1}^w,h}, \quad i = 2, \dots, M_w, \quad (14)$$

with  $\tilde{P}_{n,d_1^w,h} = z_{1,h}^{n,w}$  and  $\tilde{P}_{n,d_{i-1}^w,h}$  is the smoothed value from the previous occurrence of the same weekday type. Repeating this procedure for all  $h = 1, \dots, H$  yields the empirical load profile  $\tilde{P}_{n,d_i^w} = (\tilde{P}_{n,d_i^w,1}, \dots, \tilde{P}_{n,d_i^w,H})^\top$ .

### 3.1.4. Locally Estimated Scatterplot Smoothing (LOESS)

Locally Estimated Scatterplot Smoothing (LOESS) is a nonparametric regression method that estimates a smooth function by fitting a weighted local polynomial model in small neighborhoods of the data, rather than assuming a global functional form (Cleveland, 1979, p.829-831). Its flexibility handles time series with complex nonlinear structures.

#### 3.1.4.1. Stage 1 smoothing

For each node  $n$ , LOESS is applied to the set of pairs  $\{(t, y_{n,t})\}_{t=1}^T$ . For each query point  $t_0 \in \{1, \dots, T\}$ , the smoothed value  $\tilde{y}_{n,t_0}$  is obtained by solving a locally weighted least-squares problem. Let  $\mathcal{N}(t_0)$  denote the set of the  $\lfloor \alpha_{\text{LOESS}} \cdot T \rfloor$  time steps nearest to  $t_0$ , where  $\alpha_{\text{LOESS}} \in (0, 1)$  is the span parameter controlling the proportion of the total  $T$  observations used in each local fit, and  $\lfloor \cdot \rfloor$  denotes the floor function. The local neighborhood radius at  $t_0$  is (Cleveland, 1979, p.829-836)

$$\delta(t_0) = \max_{\tau \in \mathcal{N}(t_0)} |\tau - t_0|, \quad (15)$$

where  $\delta(t_0)$  denotes the distance from  $t_0$  to the furthest neighbor in  $\mathcal{N}(t_0)$ . Each observation  $y_{n,\tau}$  is assigned a weight according to the tricube kernel (Cleveland, 1979, p.831)

$$u_\tau(t_0) = \left(1 - \left|\frac{\tau - t_0}{\delta(t_0)}\right|^3\right)^3 \cdot \mathbf{1}\{|\tau - t_0| \leq \delta(t_0)\}, \quad (16)$$

where  $u_\tau(t_0)$  denotes the weight assigned to observation  $\tau$  when estimating the smoothed value at  $t_0$ , and  $\mathbf{1}\{\cdot\}$  is the indicator function which equals 1 if the condition holds and 0 otherwise. Observations closer to  $t_0$  receive larger weights, while those outside  $\mathcal{N}(t_0)$  receive weight zero. The locally weighted linear regression problem is then solved at each query point  $t_0$  (Cleveland, 1979, p.834)

$$\min_{\beta_0, \beta_1} \sum_{\tau=1}^T u_\tau(t_0) (y_{n,\tau} - \beta_0 - \beta_1 \tau)^2, \quad (17)$$

where  $\beta_0$  is the local intercept and  $\beta_1$  is the local slope of the degree- $p=1$  local polynomial. The smoothed value at  $t_0$  is then given by the fitted value of the local model (Cleveland, 1979, p.830)

$$\tilde{y}_{n,t_0} = \hat{\beta}_0 + \hat{\beta}_1 t_0, \quad (18)$$

where  $\hat{\beta}_0$  and  $\hat{\beta}_1$  are the solutions to (17), and  $\tilde{y}_{n,t_0}$  denotes the LOESS-smoothed active power at node  $n$  and time  $t_0$ . A smaller value of  $\alpha_{\text{LOESS}}$  yields a less smooth estimate that tracks local structure more closely, whereas a larger value produces heavier smoothing. The smoothed series  $\tilde{y}_{n,t}$  is then mapped back to the three-index form,  $\tilde{P}_{n,d,h}^{(1)}$ .

### 3.1.4.2. Stage 2 smoothing

For each node  $n$ , weekday type  $w$ , and interval  $h$ , LOESS is applied to the set of pairs  $\{(i, z_{i,h}^{n,w})\}_{i=1}^{M_w}$ . The local neighborhood radius, tricube kernel, and locally weighted regression problem are defined analogously to equations (15)–(17), with the global time index  $t$  replaced by the occurrence index  $i$  and the total series length  $T$  replaced by  $M_w$ . The Stage 2 smoothed value is (Cleveland, 1979, p.830)

$$\tilde{P}_{n,d_i^w,h} = \hat{\beta}_0 + \hat{\beta}_1 i, \quad (19)$$

where  $\hat{\beta}_0$  and  $\hat{\beta}_1$  are the solutions to the locally weighted regression at query point  $i$  for the sequence  $\{z_{j,h}^{n,w}\}_{j=1}^{M_w}$ . Repeating this procedure for all  $h = 1, \dots, H$  yields the empirical load profile  $\tilde{P}_{n,d_i^w} = (\tilde{P}_{n,d_i^w,1}, \dots, \tilde{P}_{n,d_i^w,H})^\top$ .

## 3.2. Clustering models

Clustering is applied to the Stage 2 smoothed empirical load profiles  $\tilde{P}_{n,d}$  obtained from Section 3.1. For each node  $n \in \{1, \dots, N\}$ , the daily profile vectors are organized by day type  $\delta \in \{1, 2, 3\}$ , corresponding weekdays, Saturday, Sunday, respectively, where weekdays encompass Monday through Friday. Let  $\mathcal{D}^\delta$  denote the ordered set of non-holiday days of day type  $\delta$  in the study year 2016, and let  $M^\delta = |\mathcal{D}^\delta|$  denote the total number of such days. For weekdays,  $M^{\text{weekday}} \approx 260$ ; for Saturdays and Sundays,  $M^{\text{Saturday}} \approx M^{\text{Sunday}} \approx 52$ . Note that  $M^\delta$  depends on the day type. The weekday group is approximately five times larger than either weekend group because five distinct weekday types are pooled.

The clustering input for node  $n$  and day type  $\delta$  is the collection of  $M^\delta$  daily profile vectors

$$\mathcal{X}_n^\delta = \{\tilde{P}_{n,d} : d \in \mathcal{D}^\delta\}, \quad (20)$$

where  $\tilde{P}_{n,d} = (\tilde{P}_{n,d,1}, \dots, \tilde{P}_{n,d,H})^\top \in \mathbb{R}^{H=96}$  is the empirical load profiles at node  $n$  on day  $d$ , as defined in equation (4). Here  $\tilde{P}_{n,d,h}$  denotes the empirical load profiles at node  $n$ , day  $d$ , and quarter-hour interval  $h$ . Thus, each profile vector has dimension  $H = 96$ , corresponding to the full 24-hour daily cycle.

Let  $m = 1, \dots, M^\delta$  index the days within a given node and day type, and let  $d_m^\delta$  denote the  $m$ -th element of  $\mathcal{D}^\delta$ . The  $m$ -th profile vector for node  $n$  is then  $\mathbf{x}_{n,m}^\delta := \tilde{P}_{n,d_m^\delta} \in \mathbb{R}^H$ . The number of clusters is fixed at  $k = 3$ . They correspond to the three seasonal categories. These are summer, transition, and winter.

Three distance metrics, Euclidean distance, Dynamic Time Warping (DTW), and correlation-based distance, are applied. For each combination of node, day type, and distance metric, an independent clustering is performed. The clustering results are evaluated using the

Silhouette score and the Correct Classification Rate (CCR), as described in Sections 3.2.4 and 3.2.3.

### 3.2.1. K-means clustering with Euclidean distance

K-means clustering is an unsupervised partitioning algorithm that divides a dataset into  $k$  clusters by minimizing within-cluster variance (Aggarwal and Reddy, 2014, p. 89). For a fixed node  $n$  and day type  $\delta$ , the algorithm groups the set  $\mathcal{X}_n^\delta$  into clusters  $\mathcal{C} = \{\mathcal{C}_1, \dots, \mathcal{C}_k\}$  by minimizing (Aggarwal and Reddy, 2014, p. 89)

$$\min_{\mathcal{C}_1, \dots, \mathcal{C}_k} \sum_{\ell=1}^k \sum_{\mathbf{x} \in \mathcal{C}_\ell} \|\mathbf{x} - \boldsymbol{\mu}_\ell\|^2, \quad (21)$$

where  $\boldsymbol{\mu}_\ell \in \mathbb{R}^H$  denotes the centroid of cluster  $\mathcal{C}_\ell$ , defined as the arithmetic mean of all profiles assigned to it,

$$\boldsymbol{\mu}_\ell = \frac{1}{|\mathcal{C}_\ell|} \sum_{\mathbf{x} \in \mathcal{C}_\ell} \mathbf{x}, \quad (22)$$

and  $\|\cdot\|$  denotes the Euclidean norm. The objective function in equation (21) measures the total within-cluster sum of squares. Then, the algorithm seeks cluster assignments that minimize this quantity. The iterative procedure is summarized in Algorithm 1.

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**Algorithm 1** K-means clustering of daily load profiles

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- 1: Select  $k$  initial centroids  $\{\boldsymbol{\mu}_1^{(0)}, \dots, \boldsymbol{\mu}_k^{(0)}\}$  at random.
- 2: **repeat**
- 3:     Assign each profile  $\mathbf{x}_{n,m}^\delta$  to the nearest centroid:

$$c_m = \arg \min_{\ell \in \{1, \dots, k\}} \|\mathbf{x}_{n,m}^\delta - \boldsymbol{\mu}_\ell\|^2$$

- 4:     Update each centroid as the mean of its assigned profiles:

$$\boldsymbol{\mu}_\ell = \frac{1}{|\mathcal{C}_\ell|} \sum_{\mathbf{x} \in \mathcal{C}_\ell} \mathbf{x}$$

- 5: **until** cluster assignments no longer change.
- 

Here  $c_m \in \{1, \dots, k\}$  denotes the cluster assignment of the  $m$ -th profile, and the procedure is repeated with multiple random initializations ( $n_{\text{init}} = 10$ ) to reduce sensitivity to the initial centroid selection.

### 3.2.2. K-means clustering with Dynamic Time Warping (DTW)

Dynamic Time Warping (DTW) is a well known elastic distance measure. It measures the similarity between shapes in time series data. DTW finds a warping path that gives the smallest distance.

For two profile vectors  $\mathbf{x} = (x_1, \dots, x_H)^\top$  and  $\mathbf{y} = (y_1, \dots, y_H)^\top$ , the DTW distance is computed via the cumulative cost matrix  $\mathbf{C} \in \mathbb{R}^{H \times H}$ , with elements defined recursively as (Aggarwal and Reddy, 2014, p. 364)

$$C(p, q) = (x_p - y_q)^2 + \min\{C(p-1, q), C(p, q-1), C(p-1, q-1)\}, \quad p, q = 1, \dots, H, \quad (23)$$

with boundary condition  $C(1, 1) = (x_1 - y_1)^2$ , and the DTW distance given by

$$d_{\text{DTW}}(\mathbf{x}, \mathbf{y}) = \sqrt{C(H, H)}. \quad (24)$$

Here  $C(p, q)$  denotes the minimum cumulative cost of aligning the sub-sequences  $(x_1, \dots, x_p)$  and  $(y_1, \dots, y_q)$ , and  $p$  and  $q$  index the quarter-hour intervals along the two profiles being compared.

The full pairwise DTW distance matrix  $\mathbf{D}^{\text{DTW}} \in \mathbb{R}^{M^\delta \times M^\delta}$  is computed for all profiles in  $\mathcal{X}_n^\delta$ , with entry  $D_{m_1, m_2}^{\text{DTW}} = d_{\text{DTW}}(\mathbf{x}_{n, m_1}^\delta, \mathbf{x}_{n, m_2}^\delta)$ . K-means clustering is then applied to this distance matrix using centroid update following Algorithm 1 with  $k = 3$  clusters in step 3.

### 3.2.3. K-means clustering with correlation-based distance

The correlation-based distance measures the dissimilarity between two profile vectors by comparing the shapes of their daily consumption patterns, regardless of their absolute magnitudes (Aggarwal and Reddy, 2014, p. 216). For two profile vectors  $\mathbf{x}, \mathbf{y} \in \mathbb{R}^H$ , the Pearson correlation distance is defined as (Virtanen et al., 2020, p. 261-272)

$$d_{\text{corr}}(\mathbf{x}, \mathbf{y}) = 1 - \frac{(\mathbf{x} - \bar{x}) \cdot (\mathbf{y} - \bar{y})}{\|\mathbf{x} - \bar{x}\|_2 \|\mathbf{y} - \bar{y}\|_2}, \quad (25)$$

where  $\bar{x} = \frac{1}{H} \sum_{p=1}^H x_p$  and  $\bar{y} = \frac{1}{H} \sum_{p=1}^H y_p$  are the means of profiles  $\mathbf{x}$  and  $\mathbf{y}$  respectively,  $(\mathbf{x} - \bar{x}) \cdot (\mathbf{y} - \bar{y})$  denotes the dot product of the two mean-centered profile vectors, and  $\|\cdot\|_2$  denotes the Euclidean norm. The fraction in equation (25) is the Pearson correlation between  $\mathbf{x}$  and  $\mathbf{y}$  (Falcone and Albuquerque, 2004, p. 2), so that  $d_{\text{corr}}(\mathbf{x}, \mathbf{y}) \in [0, 2]$ . A value of 0 indicates perfectly positively correlated profile shapes, a value of 1 indicates no

linear relationship, and a value of 2 indicates perfectly negatively correlated shapes. This formulation focuses on the direction and shape of the daily power pattern rather than its absolute level.

The full pairwise distance matrix  $\mathbf{D}^{\text{corr}} \in \mathbb{R}^{M^\delta \times M^\delta}$  is precomputed, with entry  $D_{m_1, m_2}^{\text{corr}} = d_{\text{corr}}(\mathbf{x}_{n, m_1}^\delta, \mathbf{x}_{n, m_2}^\delta)$ . K-means clustering is applied to this distance matrix following Algorithm 1 with  $k = 3$  clusters in step 3.

### 3.2.4. Silhouette score

The Silhouette score is a metric that evaluates the quality of clustering. The method evaluates both intra-cluster and inter-cluster distances for each sample (Aggarwal and Reddy, 2014, p. 93). For the  $m$ -th profile  $\mathbf{x}_{n, m}^\delta$ , the Silhouette value is defined as (Rousseeuw, 1987, p. 56)

$$s(m) = \frac{b(m) - a(m)}{\max\{a(m), b(m)\}}, \quad (26)$$

where  $a(m)$  denotes the mean intra-cluster distance from  $\mathbf{x}_{n, m}^\delta$  to all other profiles within the same cluster  $\mathcal{C}_{c_m}$  (Rousseeuw, 1987, p. 55),

$$a(m) = \frac{1}{|\mathcal{C}_{c_m}| - 1} \sum_{\substack{\mathbf{x} \in \mathcal{C}_{c_m} \\ \mathbf{x} \neq \mathbf{x}_{n, m}^\delta}} d(\mathbf{x}_{n, m}^\delta, \mathbf{x}), \quad (27)$$

and  $b(m)$  denotes the mean distance from  $\mathbf{x}_{n, m}^\delta$  to all profiles in the nearest neighboring cluster (Rousseeuw, 1987, p. 55),

$$b(m) = \min_{\ell \neq c_m} \frac{1}{|\mathcal{C}_\ell|} \sum_{\mathbf{x} \in \mathcal{C}_\ell} d(\mathbf{x}_{n, m}^\delta, \mathbf{x}). \quad (28)$$

Here  $d(\cdot, \cdot)$  denotes the distance function corresponding to the respective clustering method in Section 3.2.1, 3.2.2, and 3.2.3. The overall clustering quality is measured by the average Silhouette score over all  $M^\delta$  profiles,

$$\bar{s} = \frac{1}{M^\delta} \sum_{m=1}^{M^\delta} s(m). \quad (29)$$

The average Silhouette score  $\bar{s} \in [-1, 1]$ . The values close to 1 indicate well-separated clusters, while values near 0 indicate overlapping clusters. Whereas, negative values suggest misclassified profiles.

### 3.2.5. Hungarian algorithm

To assess the correspondence between the obtained clusters and the three seasonal categories, each cluster must be matched to a season label. Since cluster indices are arbitrary and carry no inherent ordering, a label assignment is required before any classification accuracy can be computed.

Each daily profile  $\mathbf{x}_{n,m}^\delta$  is assigned a true season label  $s_m \in \{0, 1, 2\}$ , where 0 = winter, 1 = transition, and 2 = summer, according to the season of day  $d_m^\delta$ . Let  $c_m \in \{0, 1, 2\}$  denote the cluster assignment of the  $m$ -th profile produced by the clustering algorithm. The optimal assignment is found by solving (Goldberger and Tassa, 2008, p. 4); (Kuhn, 1955, p. 89-96)

$$\hat{\pi} = \arg \max_{\pi \in S_3} \frac{1}{M^\delta} \sum_{m=1}^{M^\delta} \mathbf{1}\{\pi(c_m) = s_m\}, \quad (30)$$

where  $S_3$  denotes the set of all permutations of  $\{0, 1, 2\}$ ,  $\pi : \{0, 1, 2\} \rightarrow \{0, 1, 2\}$  is a bijection mapping cluster indices to season labels, and  $\mathbf{1}\{\cdot\}$  is the indicator function that equals 1 if its argument is true and zero otherwise. The optimal permutation  $\hat{\pi}$  maximizes the proportion of profiles correctly assigned to their true season under the mapping. Since  $|S_3| = 3! = 6$ , the optimization is solved by exhaustive enumeration over all six possible assignments, which is computationally trivial for  $k = 3$  clusters.

### 3.2.6. Correct Classification Rate (CCR)

The CCR is a standard classification accuracy measure defined as the proportion of correctly classified observations (Sokolova and Lapalme, 2009, p. 429-430). The CCR measures the proportion of daily profiles correctly assigned to their corresponding seasonal category under the optimal label assignment  $\hat{\pi}$  obtained in Section 3.2.5. It is defined as

$$\text{CCR} = \frac{1}{M^\delta} \sum_{m=1}^{M^\delta} \mathbf{1}\{\hat{\pi}(c_m) = s_m\}, \quad (31)$$

where  $c_m$  is the cluster assignment of the  $m$ -th profile,  $s_m$  is the true season label of day  $d_m^\delta$ , and  $\hat{\pi}$  is the optimal permutation as defined in equation (30). The CCR takes values in  $[0, 1]$ , where a value of 1 indicates perfect seasonal separation.

## 3.3. Multivariate Analysis of Variance (MANOVA)

Multivariate Analysis of Variance (MANOVA) generalizes univariate Analysis of Variance (ANOVA) to cases involving multiple dependent variables rather than a single outcome

measure (Rencher and Christensen, 2012, p. 169). A two-way MANOVA extends this further by simultaneously testing the effects of two categorical factors on a multivariate response vector. In this study, MANOVA tests to determine whether the empirical load profiles differ across seasons and day types.

Two MANOVA tests are conducted. MANOVA Test A is applied directly to the smoothed empirical load profiles  $\tilde{\mathbf{P}}_{n,d}$ . MANOVA Test B is applied to residual profiles. The residual is calculated as the point-wise difference between each empirical profile and the corresponding BDEW SLP. Both tests are two-way MANOVA with season and day type as fixed factors. The season effect tests whether mean empirical load profiles vary across the three BDEW seasons. The day type effect examines whether profiles differ across day categories. Additionally, the interaction between season and day type is conducted. This interaction tests whether seasonal changes in profile shape differ across day types. Each daily profile is treated as a 96-dimensional response vector. Each element represents the active power at one quarter-hourly interval. The Pillai's trace statistic is used.

### 3.3.1. MANOVA assumptions

MANOVA is based on four main assumptions (Tabachnick and Fidell, 2014, p. 293-295).

- **Multivariate normality:** Within each group, the data should follow a multivariate normal distribution. This means the dependent variables are normally distributed together across all dimensions.
- **Homogeneity of covariance matrices:** The covariance matrices must be the same for all groups, so the variance and relationships between dependent variables are consistent.
- **Independence of observations:** Each data point (observation vector) must be collected independently.
- **Fixed and balanced factors:** The grouping factors need to be fixed and categorical.

### 3.3.2. Two-way MANOVA

For a fixed node  $n$  and smoothing method, let  $\mathbf{Z}_{r,s,\delta} \in \mathbb{R}^{96}$  denote the response vector of the  $r$ -th daily observation within group  $(s, \delta)$ , where  $r = 1, \dots, N_{s,\delta}$  indexes the non-holiday days belonging to season  $s$  and day type  $\delta$ .  $N_{s,\delta}$  denotes the number of such days.  $s \in \{0, 1, 2\}$  denotes the season level, where 0 = winter, 1 = transition, and 2 = summer.  $\delta \in \{1, 2, 3\}$  denotes the day type level. The node index  $n$  is suppressed for notational brevity, as each test is conducted independently per node.



The two-way MANOVA model for each observation is (Rencher and Christensen, 2012, p. 201)

$$\mathbf{Z}_{r,s,\delta} = \boldsymbol{\mu} + \boldsymbol{\alpha}_s + \boldsymbol{\beta}_\delta + \boldsymbol{\gamma}_{s,\delta} + \boldsymbol{\varepsilon}_{r,s,\delta}, \quad (32)$$

where  $\boldsymbol{\mu} \in \mathbb{R}^{96}$  denotes the overall population mean vector,  $\boldsymbol{\alpha}_s$  denotes the main effect of season  $s$ ,  $\boldsymbol{\beta}_\delta$  denotes the main effect of day type  $\delta$ ,  $\boldsymbol{\gamma}_{s,\delta}$  denotes the season-by-day-type interaction effect, and  $\boldsymbol{\varepsilon}_{r,s,\delta} \sim \mathcal{N}_{96}(\mathbf{0}, \boldsymbol{\Sigma})$  denotes the error vector, assumed independently and identically distributed with a common  $96 \times 96$  covariance matrix  $\boldsymbol{\Sigma}$ .

Let  $\boldsymbol{\mu}_{s,\delta} = \mathbb{E}[\mathbf{Z}_{r,s,\delta}]$  denote the population mean vector for group  $(s, \delta)$ . The marginal mean vectors are defined as

$$\bar{\boldsymbol{\mu}}_s = \frac{1}{3} \sum_{\delta=1}^3 \boldsymbol{\mu}_{s,\delta}, \quad \bar{\boldsymbol{\mu}}_\delta = \frac{1}{3} \sum_{s=1}^3 \boldsymbol{\mu}_{s,\delta}, \quad \bar{\boldsymbol{\mu}} = \frac{1}{9} \sum_{s=1}^3 \sum_{\delta=1}^3 \boldsymbol{\mu}_{s,\delta}, \quad (33)$$

where  $\bar{\boldsymbol{\mu}}_s$  denotes the marginal mean vector for season  $s$  averaged over all three day types,  $\bar{\boldsymbol{\mu}}_\delta$  denotes the marginal mean vector for day type  $\delta$  averaged over all three seasons, and  $\bar{\boldsymbol{\mu}}$  denotes the overall grand mean vector.

### 3.3.3. Sum of Squares and Cross-Products (SSCP) matrices

Let  $\bar{\mathbf{Z}}_{s,\delta} = \frac{1}{N_{s,\delta}} \sum_{r=1}^{N_{s,\delta}} \mathbf{Z}_{r,s,\delta}$  denote the sample mean vector for group  $(s, \delta)$ , and let  $\bar{\mathbf{Z}}_{s\cdot}$ ,  $\bar{\mathbf{Z}}_{\cdot\delta}$ , and  $\bar{\mathbf{Z}}$  denote the corresponding marginal and grand sample mean vectors. The error SSCP matrix is (Rencher and Christensen, 2012, p. 170-173)

$$\mathbf{E} = \sum_{s=1}^3 \sum_{\delta=1}^3 \sum_{r=1}^{N_{s,\delta}} \left( \mathbf{Z}_{r,s,\delta} - \bar{\mathbf{Z}}_{s,\delta} \right) \left( \mathbf{Z}_{r,s,\delta} - \bar{\mathbf{Z}}_{s,\delta} \right)^\top, \quad (34)$$

with error degrees of freedom  $\nu_E = N_{\cdot\cdot} - 9$ , where  $N_{\cdot\cdot} = \sum_{s,\delta} N_{s,\delta}$  is the total number of observations. For each effect, the hypothesis SSCP matrix  $\mathbf{H}$  is defined as follows (Rencher and Christensen, 2012, p. 202-203). For the season effect:

$$\mathbf{H}^{(S)} = \sum_{s=1}^3 N_{s\cdot} \left( \bar{\mathbf{Z}}_{s\cdot} - \bar{\mathbf{Z}} \right) \left( \bar{\mathbf{Z}}_{s\cdot} - \bar{\mathbf{Z}} \right)^\top, \quad \nu_H^{(S)} = 2, \quad (35)$$

where  $N_{s\cdot} = \sum_{\delta=1}^3 N_{s,\delta}$ . For the day type effect:

$$\mathbf{H}^{(\delta)} = \sum_{\delta=1}^3 N_{\cdot\delta} \left( \bar{\mathbf{Z}}_{\cdot\delta} - \bar{\mathbf{Z}} \right) \left( \bar{\mathbf{Z}}_{\cdot\delta} - \bar{\mathbf{Z}} \right)^\top, \quad \nu_H^{(\delta)} = 2, \quad (36)$$

where  $N_{\cdot\delta} = \sum_{s=1}^3 N_{s,\delta}$ . For the season-by-day-type interaction:

$$\mathbf{H}^{(S \times \delta)} = \sum_{s=1}^3 \sum_{\delta=1}^3 N_{s,\delta} \left( \bar{\mathbf{Z}}_{s,\delta} - \bar{\mathbf{Z}}_{s\cdot} - \bar{\mathbf{Z}}_{\cdot\delta} + \bar{\mathbf{Z}} \right) \left( \bar{\mathbf{Z}}_{s,\delta} - \bar{\mathbf{Z}}_{s\cdot} - \bar{\mathbf{Z}}_{\cdot\delta} + \bar{\mathbf{Z}} \right)^\top, \quad \nu_H^{(S \times \delta)} = 4. \quad (37)$$

For each effect, Pillai's trace  $V^{(s)}$  is computed by substituting the respective  $\mathbf{H}$  and  $\mathbf{E}$  into equation (38), with  $s = \min(\nu_H, p)$ .

### 3.3.4. Test statistic: Pillai's trace

Each hypothesis is evaluated using Pillai's trace. For a given effect, Pillai's trace is given as (Rencher and Christensen, 2012, p. 179)

$$V^{(s)} = \text{tr}[(\mathbf{E} + \mathbf{H})^{-1}\mathbf{H}] = \sum_{\ell=1}^s \frac{\lambda_{\ell}}{1 + \lambda_{\ell}}, \quad (38)$$

where  $\mathbf{H}$  denotes the SSCP matrix for the effect under test,  $\mathbf{E}$  denotes the error SSCP matrix,  $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_s > 0$  are the ordered eigenvalues of  $\mathbf{E}^{-1}\mathbf{H}$ ,  $s = \min(\nu_H, p)$  with  $\nu_H$  denoting the hypothesis degrees of freedom, and  $p = 96$  the dimension of the response. The statistic  $V^{(s)}$  lies in  $[0, s]$ , with larger values indicating stronger evidence against the null hypothesis.

Pillai's trace is often preferred to other test statistics because it is more reliable, especially when the data do not meet the multivariate normality assumption.

An approximate  $F$ -statistic for Pillai's trace is given by (Rencher and Christensen, 2012, p. 180):

$$F = \frac{(2N + s + 1) \cdot V^{(s)}}{(2m + s + 1)(s - V^{(s)})}, \quad (39)$$

which is approximately distributed as  $F_{s(2m+s+1), s(2N+s+1)}$  under the null hypothesis, where  $m = \frac{1}{2}(|\nu_H - p| - 1)$  and  $N = \frac{1}{2}(\nu_E - p - 1)$ , with  $\nu_E$  denoting the error degrees of freedom. The corresponding  $p$ -values are extracted from this  $F$ -approximation. Each null hypothesis is rejected at significance level  $\alpha_{\text{sig}}$ , i.e.  $H_0$  is rejected if the  $p$ -value satisfies  $p < \alpha_{\text{sig}}$ .

### 3.3.5. Hypotheses

The following hypotheses are tested using Pillai's trace  $V^{(s)}$  as defined in equation (38).

**Season effect:**

$$\begin{aligned} H_0^{(S)} : \bar{\boldsymbol{\mu}}_{s_1} &= \bar{\boldsymbol{\mu}}_{s_2} = \bar{\boldsymbol{\mu}}_{s_3} \\ H_1^{(S)} : \text{not all } \bar{\boldsymbol{\mu}}_{s_1} &= \bar{\boldsymbol{\mu}}_{s_2} = \bar{\boldsymbol{\mu}}_{s_3} \end{aligned}$$

**Day type effect:**

$$H_0^{(\delta)} : \bar{\mu}_{\delta_1} = \bar{\mu}_{\delta_2} = \bar{\mu}_{\delta_3}$$

$$H_1^{(\delta)} : \text{not all } \bar{\mu}_{\delta_1} = \bar{\mu}_{\delta_2} = \bar{\mu}_{\delta_3}$$

**Season  $\times$  day type interaction:**

$$H_0^{(S \times \delta)} : \mu_{s,\delta} - \bar{\mu}_s - \bar{\mu}_\delta + \bar{\mu} = 0 \quad \forall s, \delta$$

$$H_1^{(S \times \delta)} : \exists (s, \delta) \text{ such that } \mu_{s,\delta} - \bar{\mu}_s - \bar{\mu}_\delta + \bar{\mu} \neq 0$$

Each hypothesis is evaluated independently via the  $F$ -approximation in equation (18), and the corresponding  $p$ -values are reported for each node and smoothing method.

## 4. Results

In this section, the time series of active power for the five selected nodes are analyzed and interpreted by the statistical methods described in the previous section. The results are discussed in three parts. Section 4.1 presents the empirical load profile analysis, which includes the two-stage smoothing process and compares the average empirical load profiles with the BDEW SLPs. Section 4.2 explains the K-means clustering results using different distance metrics and smoothing models. Section 4.3 discusses the MANOVA results. It examines how season, day type, and their interaction affect the empirical load profiles and the residual profiles.

### 4.1. Empirical load profile analysis

This section presents the results of the empirical load profile analysis applied to the five selected nodes. The analysis is structured in two parts. Section 4.1.1 introduces the sequential two-stage smoothing pipeline and assesses the effect of holiday removal. Section 4.1.2 presents the mean empirical load profiles and compares them with the BDEW SLPs. The results in this section provide a foundation for clustering (Section 4.2) and MANOVA analysis (Section 4.3).

#### 4.1.1. Sequential smoothing

The active power time series at each node is very noisy, as shown in Figure 26 in the Appendix A.1. Figure 26 displays the raw active power time series for all five nodes in

2016. There are two agricultural nodes (Nodes 1 and 12) and three household nodes (Nodes 8, 11, and 13). Node 1 shows consistently high variability throughout the year across all seasons. The signal shows no meaningful reduction in activity across any season. This suggests a continuous operational nature of agricultural consumers. Node 12 behaves differently from the other nodes. This node maintains a higher baseline and regularly produces large consumption spikes throughout the transition and summer periods. The transition period on the right side exhibits the most intense activity. The signal suddenly drops to near zero and remains there for the rest of the year.

The household nodes display varying degrees of seasonal suppression. Node 8 has a clear seasonal pattern. In summer, low consumption falls to nearly 0.0 p.u. In addition, there are fewer high amplitude spikes compared to other seasons. This activity increases again during the transition and winter periods. This pattern matches a household node with lower demand in warmer months. Node 11 has a seasonal pattern similar to Node 8, but the differences between seasons are more noticeable. The node shows high variability with frequent during the winter periods. During the summer, the signal shows only a few small spikes. This clear seasonal difference suggests the household uses much less power when the weather is warm. Node 13 displays moderately active and variable behavior during winter and transition periods. A degree of seasonal suppression is seen during summer. Activity increases during the right transition and winter periods and returns to a pattern similar to the start of the year.

The data shows that there are two different sources of variability. The first is within-day noise. This noise refers to high frequency fluctuations within a single day. As brief spikes in demand can change the shape of daily profiles. Another noise appears across weeks. The load shapes differ systematically across repeated instances of the same weekday type.

The sequential two-stage pipeline addresses these issues separately. In Stage 1, the annual time series is smoothed to reduce daily noise. The series is then split into daily profiles. In Stage 2, an additional smoothing is applied across the same weekdays to reduce variability between weeks. Public holidays are identified and removed between the two stages. This step prevents anomalous consumption patterns in the representative profiles derived in Stage 2.

#### **4.1.1.1. Stage 1: Cross-day noise reduction**

In Stage 1 smoothing, the dataset for the entire year is treated as a single continuous time series consisting of approximately 35,136 quarter-hourly observations. The effective window of 8 time steps (2 hours) is selected to balance noise and shape preservation. Quarter-hourly power time series typically persist for 1 to 4 time steps (15 to 60 minutes). A 2-hour window is therefore wide enough to reduce noise while maintaining its characteristic shape without distortion. The shorter window (e.g. 30 minutes) leaves high

frequency noise, whereas the longer window (e.g. 4–6 hours) begins to blur transitions between load values. Consequently, the output of Stage 1 is a smoothed representation of the full-year series.

Table 1 summarizes the parameters applied to each smoothing method.

Table 1: Stage 1 smoothing parameters.

| Method                | Parameter                | Value    | Derivation                                |
|-----------------------|--------------------------|----------|-------------------------------------------|
| Moving Average        | <code>window_size</code> | 8        | $8 \times 15 \text{ min} = 2 \text{ hrs}$ |
| Moving Median         | <code>window_size</code> | 8        | Same as MA; robust to outlier spikes      |
| LOESS                 | <code>fraction</code>    | 0.000227 | $8 / 35,136$                              |
| Exponential smoothing | $\alpha_{expo}$          | 0.22     | $\alpha_{expo} = 2/(N + 1)$ , $N = 8$     |

Figures 3, 4, and 5 display the Stage 1 smoothing output for three representative days across all five nodes, which are a weekday (2016-05-09), a Saturday (2016-08-06), and a Sunday (2016-06-19), respectively. These dates are not selected for any purpose but serve to illustrate how each smoother behaves across different day types.

The Moving Average and Moving Median produce nearly identical outputs across all three days and all five nodes. Their results closely follow the shape of the original data while smoothing out short duration spikes. The signal itself is fairly stable with few extreme outliers. The Moving Median suppresses the outliers slightly more effectively than the Moving Average. This is because the Moving Median takes the middle value of a window, which is less affected by a single extreme point.

Exponential smoothing introduces a visible phase lag throughout all nodes and all three days. The smoothed curve usually lags behind the original signal. This behavior occurs because Exponential smoothing considers each smoothed value solely on past observations. The lag is most apparent at Node 1 on the weekday and at Node 12 on the Saturday. However, this effect does not fundamentally distort the overall daily shape.

LOESS performs differently from the other three methods when the signal contains very short spikes. On Node 13, LOESS overshoots the original peak amplitude across all three days. This is most clearly seen in the weekday plot, where LOESS reconstructs a spike around time step 10 with a higher amplitude than the original signal. This situation happens because LOESS fits a smooth curve through a local neighborhood of points. Then the fitted curve is pulled upward at the spike and overshoots the original value. On the other hand, Moving Average, Moving Median, and Exponential smoothing reduce the size of spikes. The LOESS overshoot is also visible in Node 12 on the Saturday, where the evening peak is reconstructed at approximately 0.9 p.u.

Overall, Moving Average and Moving Median are the most reliable methods in this smoothing stage. Exponential smoothing introduces a mild but consistent temporal lag.

LOESS performs well on smooth profiles but tends to amplify narrow spikes. This suggests that the choice of smoothing method matters little and the 2-hour window is wide enough to remove noise without distorting the overall load shape.

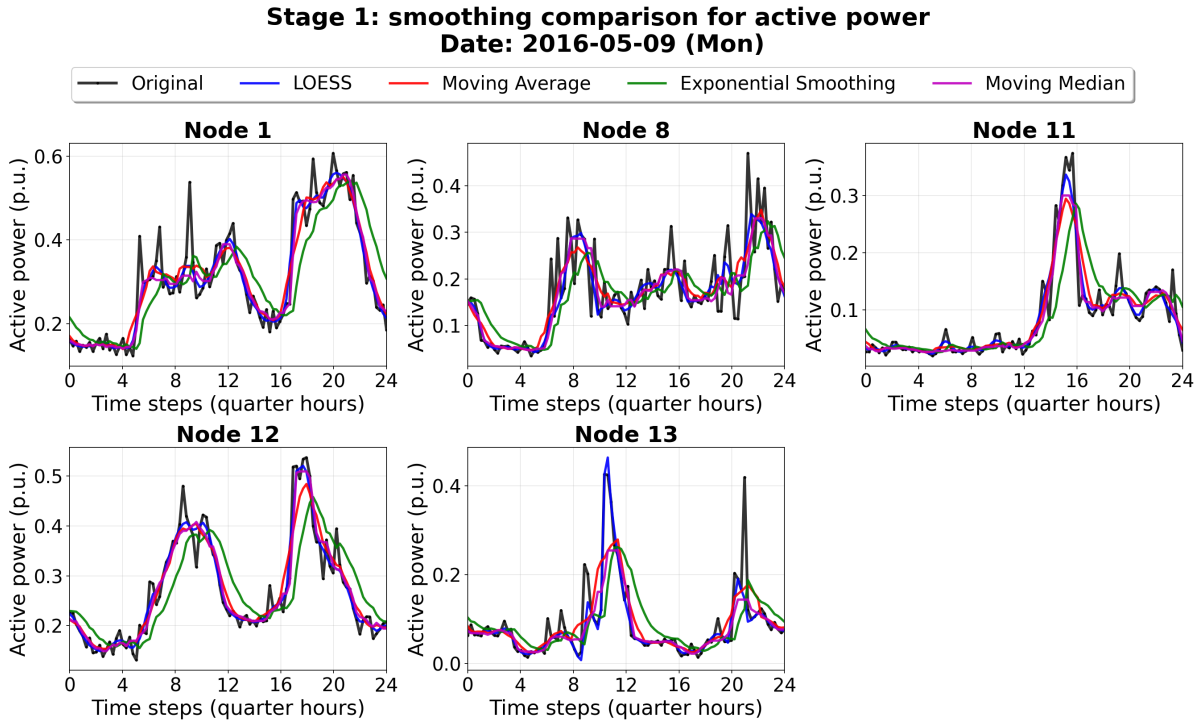


Figure 3: Stage 1 smoothing comparison across all nodes: representative weekday.

#### 4.1.1.2. Holidays removal

This step ensures that continuous series smoothing is not disrupted by gaps in the time index. The nine public holidays in Section 2.4 are removed at this step. Following the holiday removal step, the dataset is structured into weekdays, Saturdays, and Sundays. Then, the Stage 2 smoothing is performed on the data. The effect of this removal on the resulting profiles is examined in Section 4.1.1.4.

#### 4.1.1.3. Stage 2: Cross-week noise reduction

In Stage 2, the smoothed annual series from Stage 1 is split into individual daily profiles and reorganized by day type. For each combination of node and day type (Monday through Sunday), all annual occurrences of that day type are extracted (approximately 52 per year) and arranged into a matrix of shape (weeks  $\times$  96 intervals). Smoothing methods are then applied independently down each column across the approximately 52 weekly values at each fixed quarter-hourly interval. The effective window of 4 weekly occurrences is selected to balance anomaly suppression and seasonal shape preservation. It is important to note that this window of 4 does not refer to 4 consecutive calendar days. Instead, it refers to 4 consecutive occurrences of the same day type, for example, 4

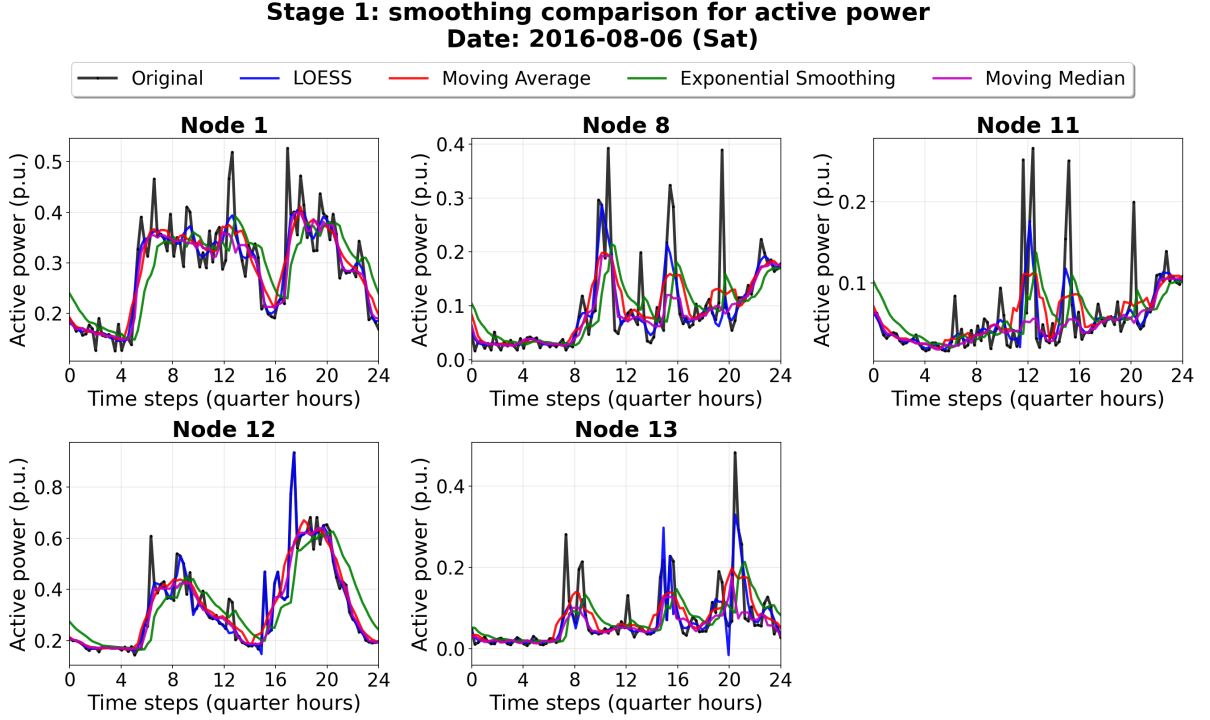


Figure 4: Stage 1 smoothing comparison across all nodes: representative Saturday.

Mondays, 4 Saturdays, and so on. Each day type is treated as an independent time axis of approximately 52 points, and the smoothing window of 4 moves along it. This means that a Monday is only ever smoothed against other Mondays, and a Sunday is only ever smoothed against other Sundays. The fluctuations in electricity demand typically persist for one to three weekly occurrences. Hence, a 4-week window is good enough to smooth these anomalies while remaining narrow relative to the seasonal cycle. Such that the seasonal transitions in the data are retained. A shorter window (e.g., 1–2 weeks) leaves week-to-week noise largely intact, whereas a longer window (e.g., 8–12 weeks) tends to average across seasons and flattens the meaningful differences between season load shapes. Consequently, the output of Stage 2 is a smoothed representative profile for each node and day type combination.

Table 2 presents the parameters applied to each smoothing method in Stage 2.

Table 2: Stage 2 smoothing parameters.

| Method                | Parameter       | Value | Derivation                                       |
|-----------------------|-----------------|-------|--------------------------------------------------|
| Moving Average        | window_size     | 4     | 4 weekly occurrences $\approx$ 4 weeks, centered |
| Moving Median         | window_size     | 4     | Same as MA; robust to anomalous weeks            |
| LOESS                 | fraction        | 0.077 | 4 / 52 weekly occurrences                        |
| Exponential smoothing | $\alpha_{expo}$ | 0.40  | $\alpha_{expo} = 2/(N + 1)$ , $N = 4$            |

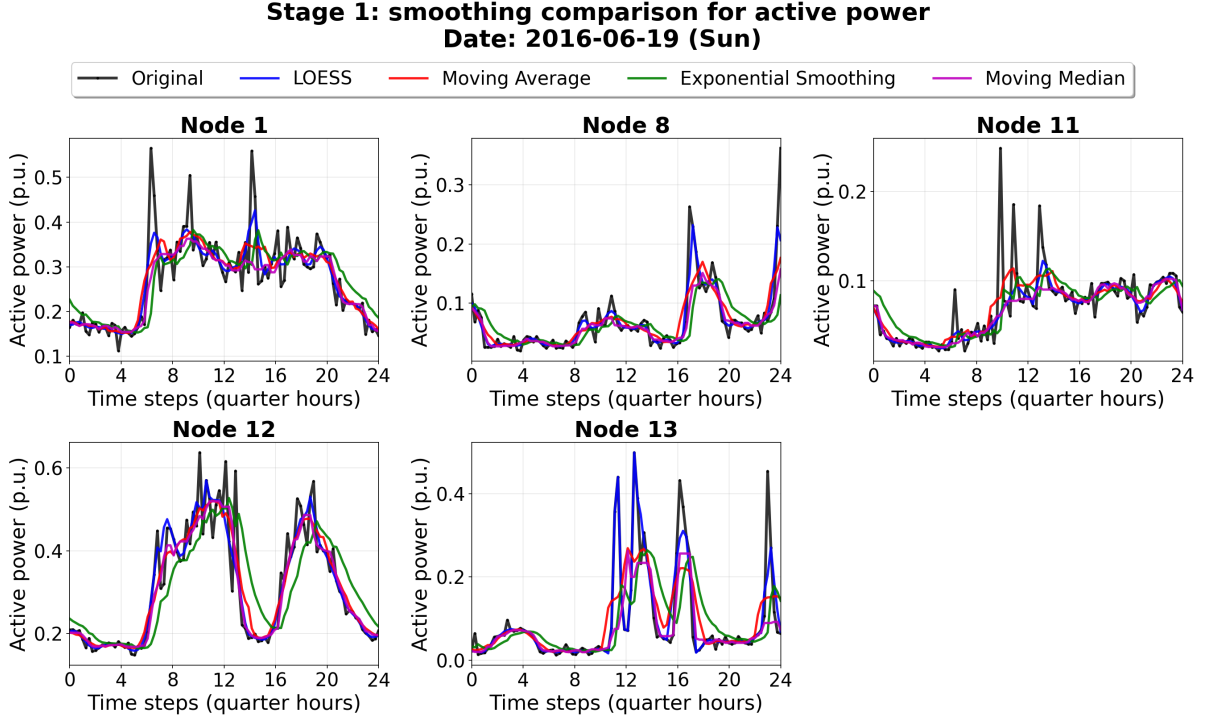


Figure 5: Stage 1 smoothing comparison across all nodes: representative Sunday.

Figures 6, 7, and 8 present the output from Stage 2 smoothing for three representative days across all five nodes, which are a weekday (2016-11-08), a Saturday (2016-05-14), and a Sunday (2016-09-25), respectively.

Moving Average and Moving Median again produce similar results across all three days and all five nodes. Both methods capture the general shape of the power time series while reducing noise between weeks. Their outputs largely overlap throughout most of the day, with only minor differences appearing at sharp transitions.

Exponential smoothing continues to show a visible phase lag across all nodes and all three days. The lag is particularly noticeable in Node 12 on the Sunday and in Node 11 on the weekday. In both cases, the morning increase in consumption appears several time steps later than it does in the raw signal. A significant issue observed in Stage 2 is that Exponential Smoothing maintains an elevated baseline during periods of low activity. The smoothed curve remains above the original data during low consumption intervals at Nodes 8 and 13 across all three days. This is because Exponential smoothing retains past periods of high consumption, even after demand decreases.

LOESS shows the most problematic behavior in Stage 2 among the four methods. On Node 12 and Node 13 in the Sunday plot, LOESS produces a smoothed curve that differs from the other three methods. The LOESS curve also lies above the active power time series for much of the day. The distortion spreads across a wider portion of the daily profile. This happens because Stage 2 operates on a much shorter time axis of approxi-



mately 52 weekly occurrences. It makes the local polynomial fit more sensitive to clusters of anomalous weeks within the neighborhood. LOESS performs better on the weekday plot, and its output is closer to Moving Average and Moving Median.

Moving Average and Moving Median are still the most reliable methods for both stages. Exponential smoothing introduces a more temporal lag and a higher baseline in Stage 2 than in Stage 1. LOESS performs better when the data is stable across weeks. However, it still tends to produce a distorted representative profile when several anomalous weeks fall close together within its local neighborhood.

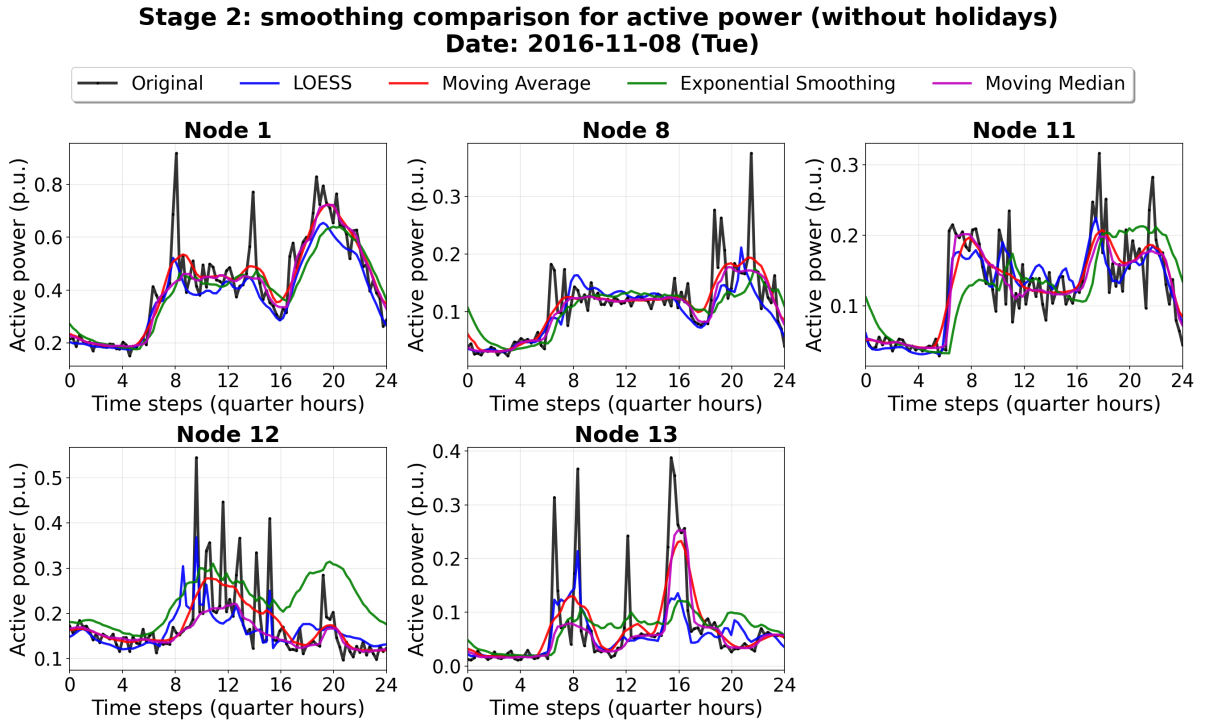


Figure 6: Stage 2 smoothing comparison across all nodes: representative weekday.

#### 4.1.1.4. Effect of holidays on empirical load profiles

Under the BDEW SLP framework in Section 2.4, public holidays are treated as Sundays regardless of the calendar day. However, the SimBench dataset shows that holidays retain their original calendar day type. For example, New Year’s Day (1 Jan 2016) is labeled as a Friday, not a Sunday. This means their anomalous consumption patterns are included in the weekday, Saturday, or Sunday profiles. To avoid introducing noise, all nine public holidays are removed from the dataset between Stage 1 and Stage 2. As a result, cross-week smoothing considers only regular day occurrences.

Table 3 lists the nine public holidays for 2016, their calendar weekday, BDEW season, and the day type category from which each is removed. The largest reduction occurs in the transition weekday category. This includes Good Friday (25 Mar, Friday), Easter Monday (28 Mar, Monday), Ascension Day (5 May, Thursday), and Unity Day (3 Oct,

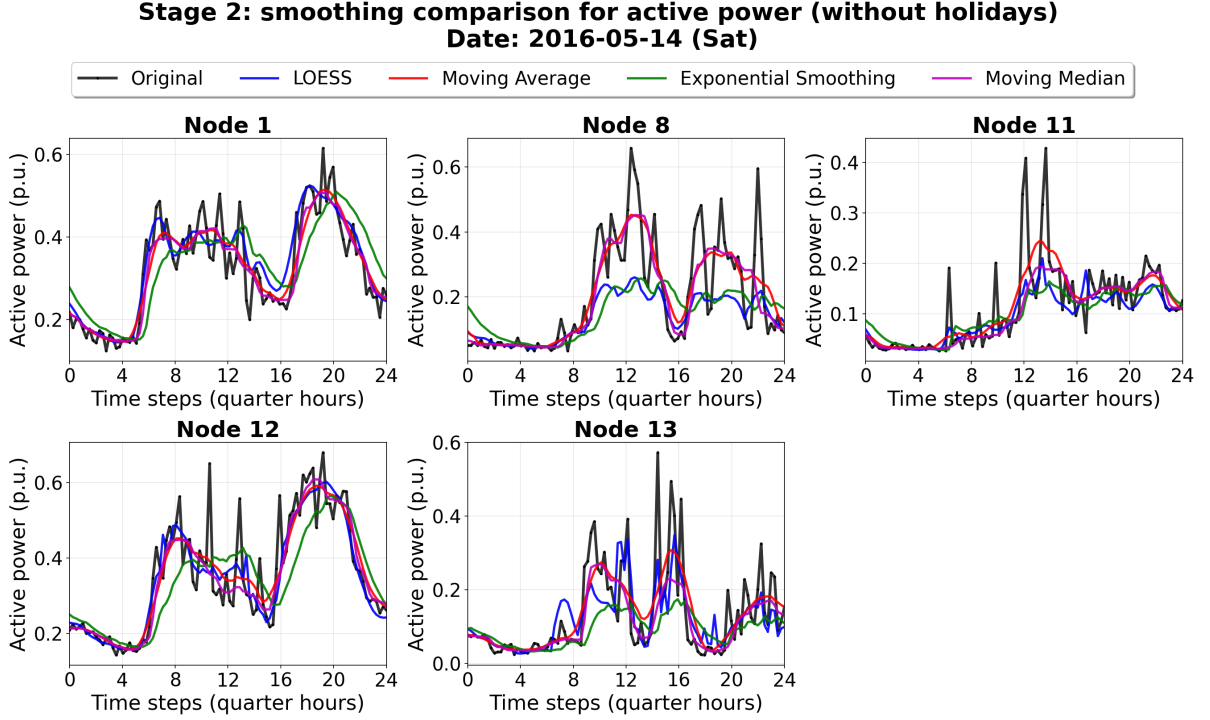


Figure 7: Stage 2 smoothing comparison across all nodes: representative Saturday.

Monday). Labor Day (1 May 2016) falls on a Sunday, therefore, it reduces the transition Sunday count rather than the weekday count. Monday (16 May 2016) falls on a Monday but lies within the BDEW summer period, so it reduces the summer weekday count by one day instead. Two days are removed from winter weekdays, which include New Year's Day (1 Jan, Friday) and Boxing Day (26 Dec, Monday). Labor Day during the transition period and Christmas Day in winter (both Sundays) are excluded from the dataset. No Saturday in 2016 coincides with any of the nine holidays, hence Saturday counts are not changed across all seasons.

Table 3: The nine public holidays used with their day of week, BDEW season classification, and the day type category.

| Date        | Name           | Day of week | BDEW season | Day type removed |
|-------------|----------------|-------------|-------------|------------------|
| 1 Jan 2016  | New Year's Day | Friday      | Winter      | Weekday          |
| 25 Mar 2016 | Good Friday    | Friday      | Transition  | Weekday          |
| 28 Mar 2016 | Easter Monday  | Monday      | Transition  | Weekday          |
| 1 May 2016  | Labor Day      | Sunday      | Transition  | Sunday           |
| 5 May 2016  | Ascension Day  | Thursday    | Transition  | Weekday          |
| 16 May 2016 | Whit Monday    | Monday      | Summer      | Weekday          |
| 3 Oct 2016  | Unity Day      | Monday      | Transition  | Weekday          |
| 25 Dec 2016 | Christmas Day  | Sunday      | Winter      | Sunday           |
| 26 Dec 2016 | Boxing Day     | Monday      | Winter      | Weekday          |

BDEW seasons: Winter = 1 November-20 March; Summer = 15 May-14 September; Transition = remainder. No Saturday coincides with any of the nine holidays in 2016.

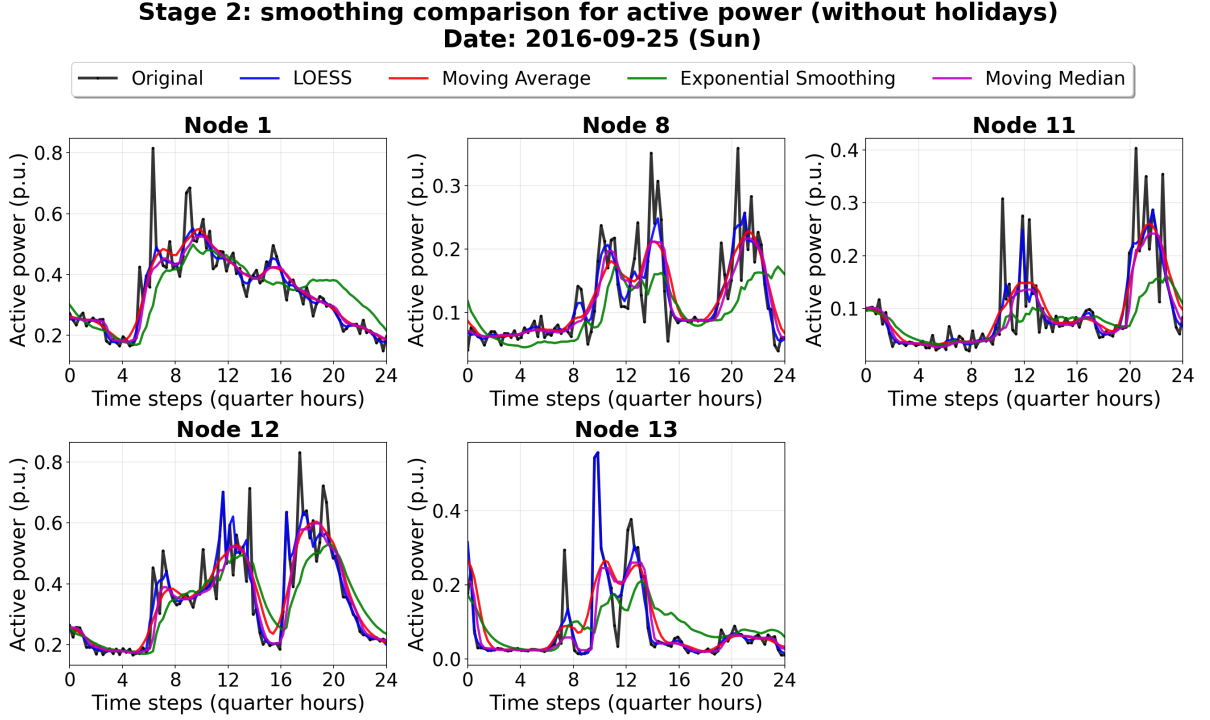


Figure 8: Stage 2 smoothing comparison across all nodes: representative Sunday.

To evaluate whether removing holidays affects the smoothed profiles, Stage 2 smoothing is performed twice independently. One run uses the Stage 1 smoothed data with public holidays included. The other uses the same data but with holidays excluded, as described in Section 4.1.1.1.

Figures 9, 10, and 11 show the Stage 2 smoothed output with holidays included for three sample days across all five nodes, which are a weekday (2016-05-09), a Saturday (2016-08-06), and a Sunday (2016-06-19).

Both the Moving Average and Moving Median methods produce results very similar to their versions without holidays. This holds regardless of the day or node. Both methods capture the general shape of the active power time series and smooth out noise between weeks. However, their outputs differ slightly at sharp transitions. The Moving Median demonstrates greater resistance to anomalous weekly values.

Exponential smoothing still shows the same phase lag and higher baseline behavior. The lag is most visible in Node 12 on Saturday and in Node 13 on the weekday. In both cases, the morning consumption ramp appears several time steps later than in the raw signal. At Nodes 8 and 13, the smoothed curve sits above the raw signal during quiet periods. This reflects the same slow exponential decay pattern observed previously.

LOESS again shows the most deviation from the other methods. The LOESS curve differs from the others and lies above the active power time series in Node 13 on the weekday

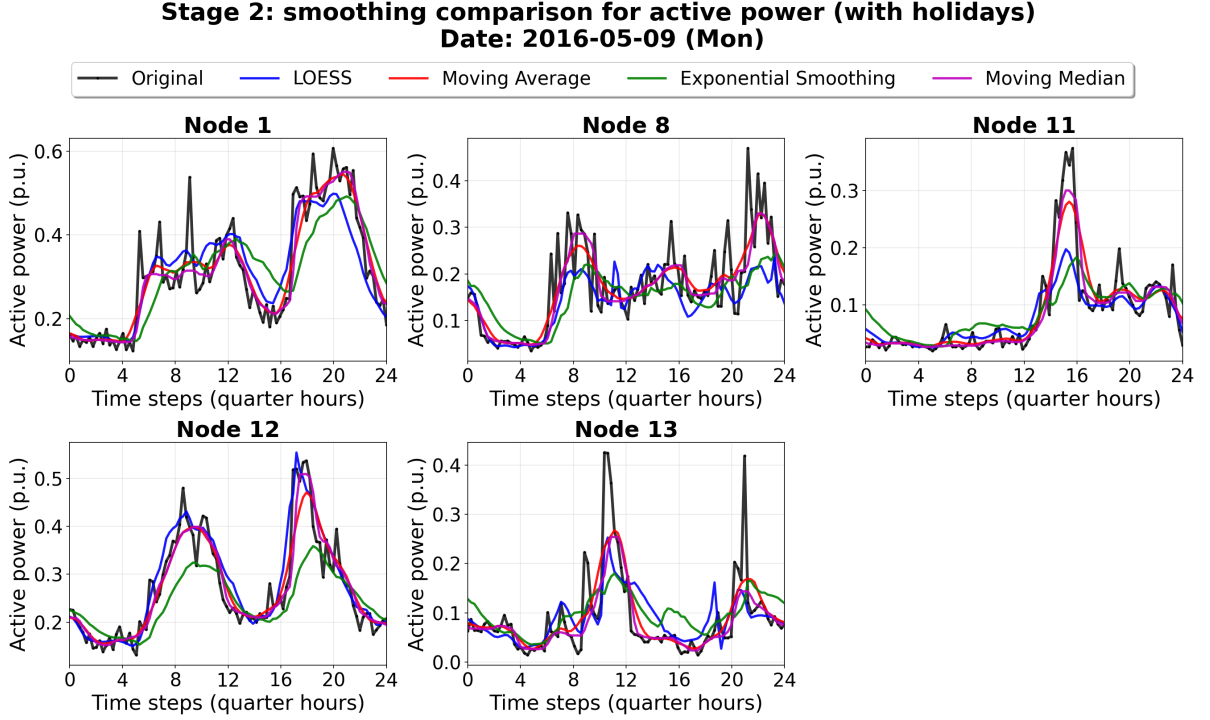


Figure 9: Stage 2 smoothing comparison across all nodes with holidays: representative weekday.

and in Node 12 on the Saturday. It again suggests the local polynomial fit is sensitive to clusters of anomalous weeks in the neighborhood.

The smoothed curves produced at Stage 2 show very little difference between the two dataset versions. In both cases, the overall profile shapes, peak timings, and baseline levels remain practically the same between the two versions. This comparison suggests that holidays have no material effect on the Stage 2 smoothed load profiles.

#### 4.1.2. Mean empirical load profiles and BDEW Standard Load Profiles (SLPs)

This section assesses the standardized reference curves and examines whether they accurately reflect observed consumption behavior in the network. The empirical load profiles in Section 4.1.1 serve as the basis for this evaluation.

A  $3 \times 3$  grid of subplots is generated for each of the five nodes and each of the four smoothing methods. Each subplot presents the mean empirical active power profile across seasons and day types. The solid lines show the empirical means for each season. The shaded regions represent the  $\pm 3\sigma$  band that represents the empirical standard deviation at each of the 96 quarterly hour intervals. The dashed lines show the BDEW SLP for the corresponding season and day type. The day-count label  $n$  shown above each subplot indicates the number of days entering the mean.

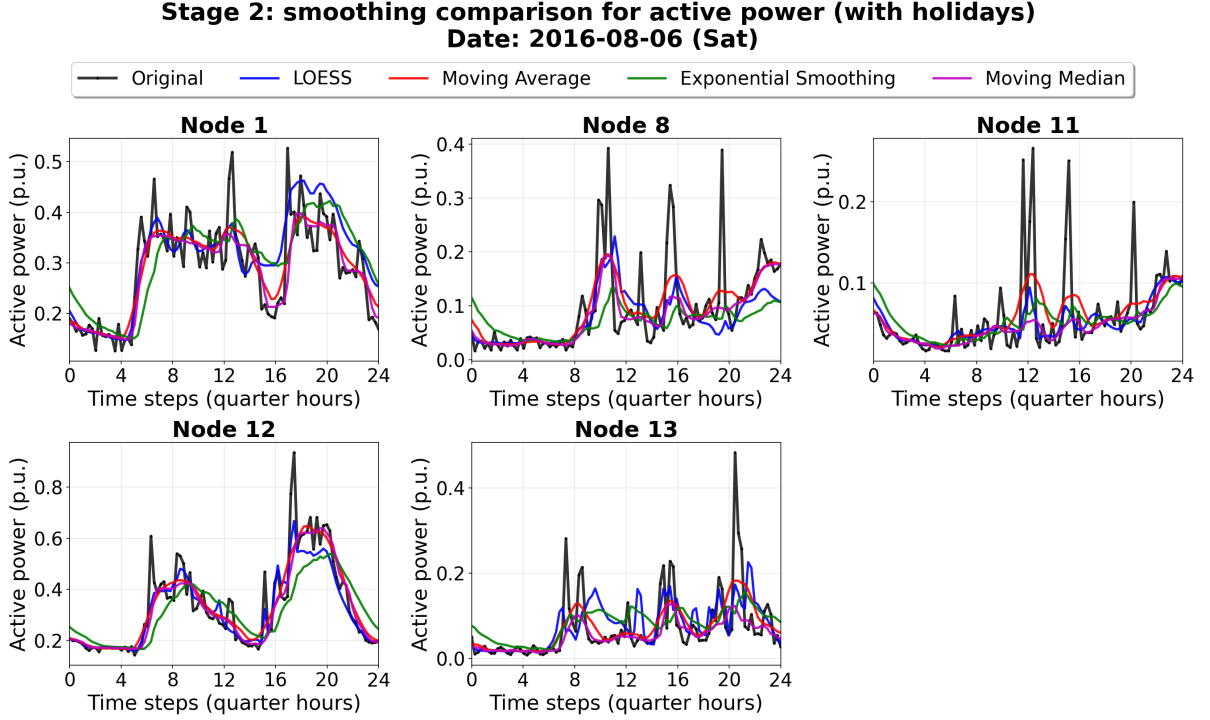


Figure 10: Stage 2 smoothing comparison across all nodes with holidays: representative Saturday.

This section focuses on two representative nodes. Node 1 (Agriculture) and Node 8 (Household) are selected as the primary representatives for discussion in this section. These two nodes are selected because they each represent one of the two consumer types in the network. The remaining nodes for Nodes 11, 12, and 13 are briefly discussed in Section 4.1.2.3 and their corresponding plots are presented in Appendix A.2.

#### 4.1.2.1. Mean empirical load profile of agriculture representative node: Node 1

Node 1 shows an agricultural load shape that remains consistent across all seasons and day types. The empirical mean is low overnight, rises to a moderate level in the morning, then peaks further in the afternoon and evening before declining again. The profile shape is similar on weekdays, Saturdays, and Sundays. Seasonal differences in level are visible. The empirical means are higher in winter and lower in summer. Transition profiles fall between these two extremes. The weekday group is by far the largest group, while each Saturday and Sunday group contains considerably fewer days per season.

There is a large constant gap between the empirical mean and the SLP for all Node 1 plots. The L2 SLP lies consistently above the empirical mean across all seasons, day types, and smoothing methods. The gap is smaller in winter, but the SLP still lies above the mean. Moreover, the SLP shape exceeds the upper uncertainty band boundary of the empirical distribution in summer and transition weekday panels. This strongly

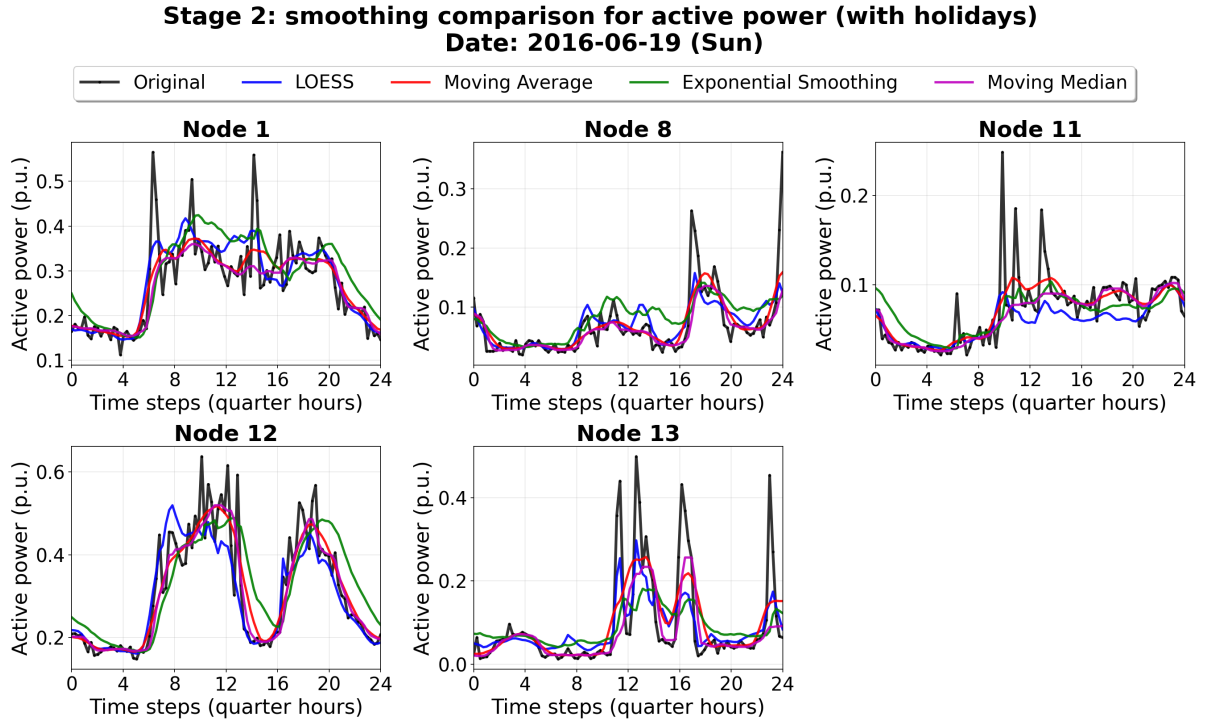


Figure 11: Stage 2 smoothing comparison across all nodes with holidays: representative Sunday.

suggests a structural overestimation of consumption at this node. The  $\pm 3\sigma$  bands are moderately wide and broadly symmetric around the mean. Weekend panels show slightly wider bands than corresponding weekday panels due to fewer observations and greater day-to-day variability in agricultural activity.

All four smoothing methods produce similar empirical mean curves. Moving Average and Moving Median profiles are nearly identical. Exponential smoothing is also very similar to both methods, with only a minor baseline elevation in some panels. LOESS produces comparable curves, likely because Node 1's weekday pattern is fairly stable.

#### 4.1.2.2. Mean empirical load profile of household representative node: Node 8

Node 8 presents a different picture from Node 1. The empirical mean is very low and nearly flat throughout the day across all seasons and day types. There are no morning, midday, or evening peaks. Seasonal differences in mean levels are small, with winter means slightly higher than summer means.

The gap between the empirical mean and the H0 SLP is more extreme than for Node 1. The H0 SLP shows sharp morning and evening peaks, while the empirical mean remains nearly flat. In most panels, the SLPs are entirely above the empirical mean and the  $\pm 3\sigma$  band, particularly in summer and transition weekday and Sunday panels. Only in a few winter cases touch or overlap the upper band. This consistent pattern strongly suggests that the H0 profile does not capture the actual load behavior at this node across all



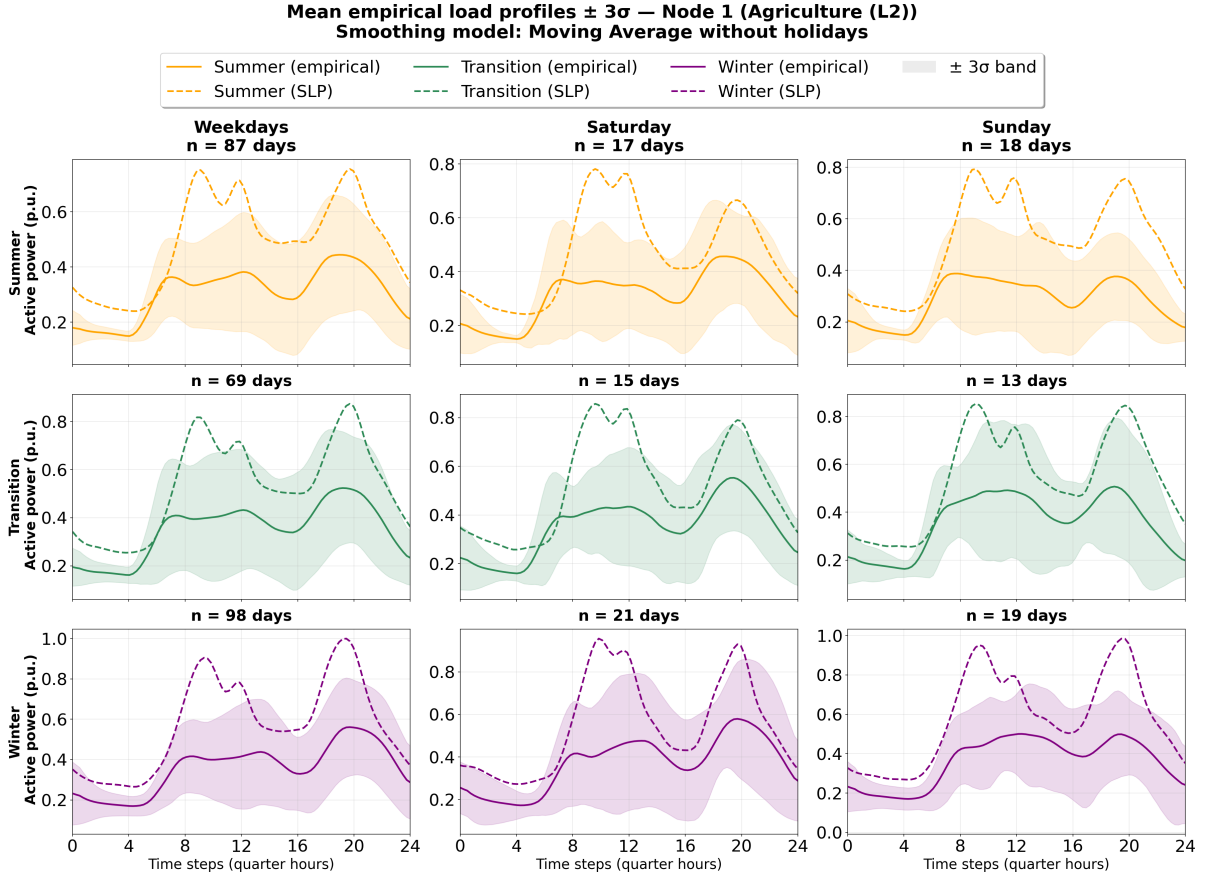


Figure 12: Mean empirical load profiles  $\pm 3\sigma$  for Node 1 (Agriculture, L2) smoothed with Moving Average (without holidays).

four smoothing methods. The characteristic double-peak shape of the SLP is simply not present in the empirical load profiles.

The uncertain bands for Node 8 are very wide relative to the mean, with the lower boundary dropping below zero in several panels. The wide bands suggest the high variability in individual household consumption.

Moving Average and Moving Median produce nearly identical mean profiles and uncertainty bands. Exponential smoothing produces a slightly elevated baseline in several panels, mostly seen in winter and transition profiles. LOESS produces wider and more irregular uncertainty bands than the other three methods. This behavior reflects LOESS's sensitivity to localized clusters of anomalous weeks. Thus, LOESS is the least appropriate smoother for this node.

#### 4.1.2.3. Mean empirical load profiles of remaining nodes

Nodes 11 and 13 have profile patterns similar to those of Node 8. The empirical means are nearly flat and remain well below the H0 SLP across all seasons, day types, and smoothing methods. The characteristic double-peak shape of the SLP is absent from both nodes. Node 13 shows the lowest consumption level among the five nodes. The

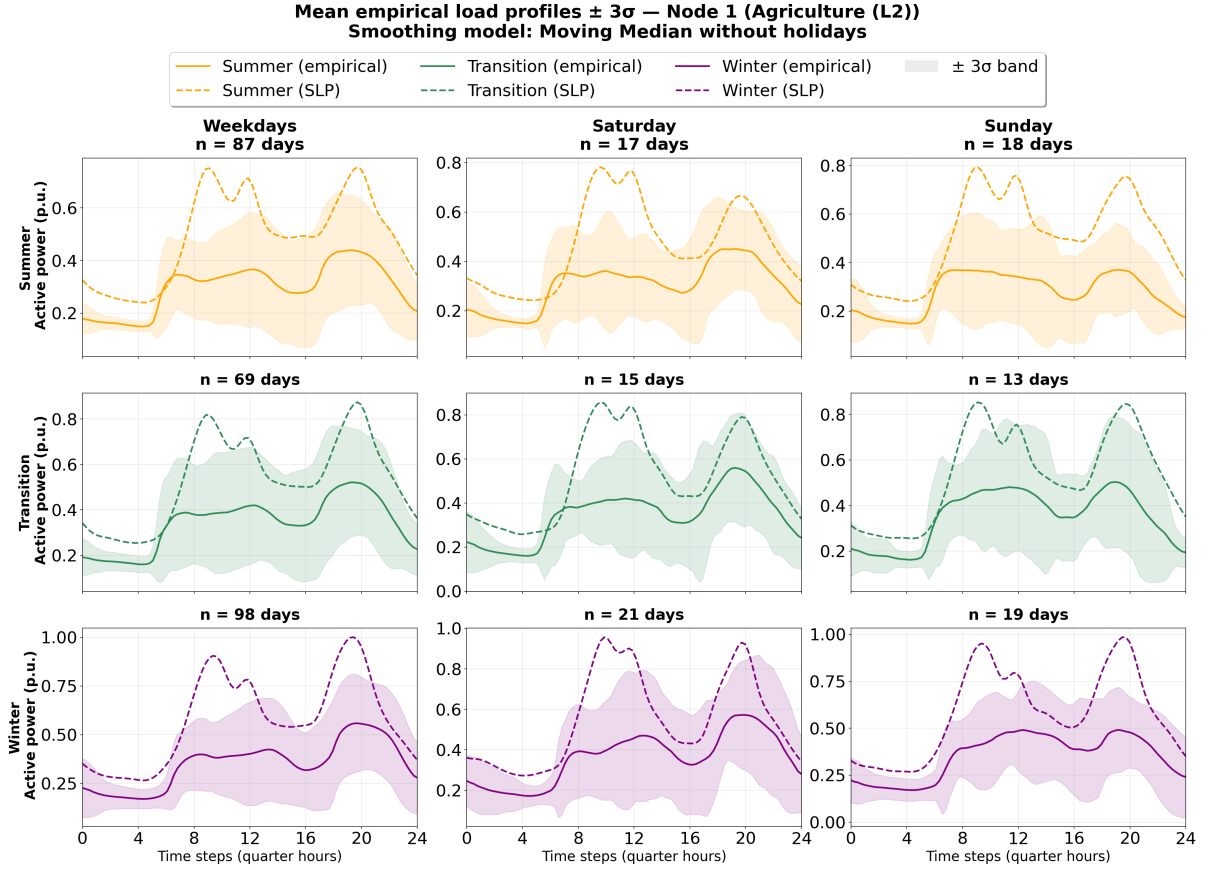


Figure 13: Mean empirical load profiles  $\pm 3\sigma$  for Node 1 (Agriculture, L2) smoothed with Moving Median (without holidays).

uncertainty bands of Node 13 are the widest in relative terms. For LOESS, the bands at Node 13 extend into negative territory in several panels. This supports that LOESS is the least suitable smoother for nodes with high variability between days. Node 11 lies between Node 8 and Node 13 in terms of consumption level and variability. Its winter profiles show slightly more intraday structure than the other two household nodes, with a rise toward the evening hours visible in the weekday and Sunday panels. Despite this, the SLP remains above the empirical mean across all panels.

Node 12 differs from Node 1 even though they belong to the same profile type. The empirical mean shows two narrow peaks. There is one in the morning and one in the evening. The L1 SLP fits this bimodal shape considerably better than the L2 SLP fits Node 1. In several panels, the SLP curve falls within or near the upper  $\pm 3\sigma$  boundary. However, the uncertainty bands are notably wide, particularly in winter. This reflects the large variability between days. LOESS produces particularly wide and irregular bands for this node. This is consistent with the Stage 2 smoothing findings.

Hence, both smoothed datasets in the plots are again approximately identical across all three remaining nodes. The plots for Nodes 11, 12, and 13 are provided in Appendix A.2



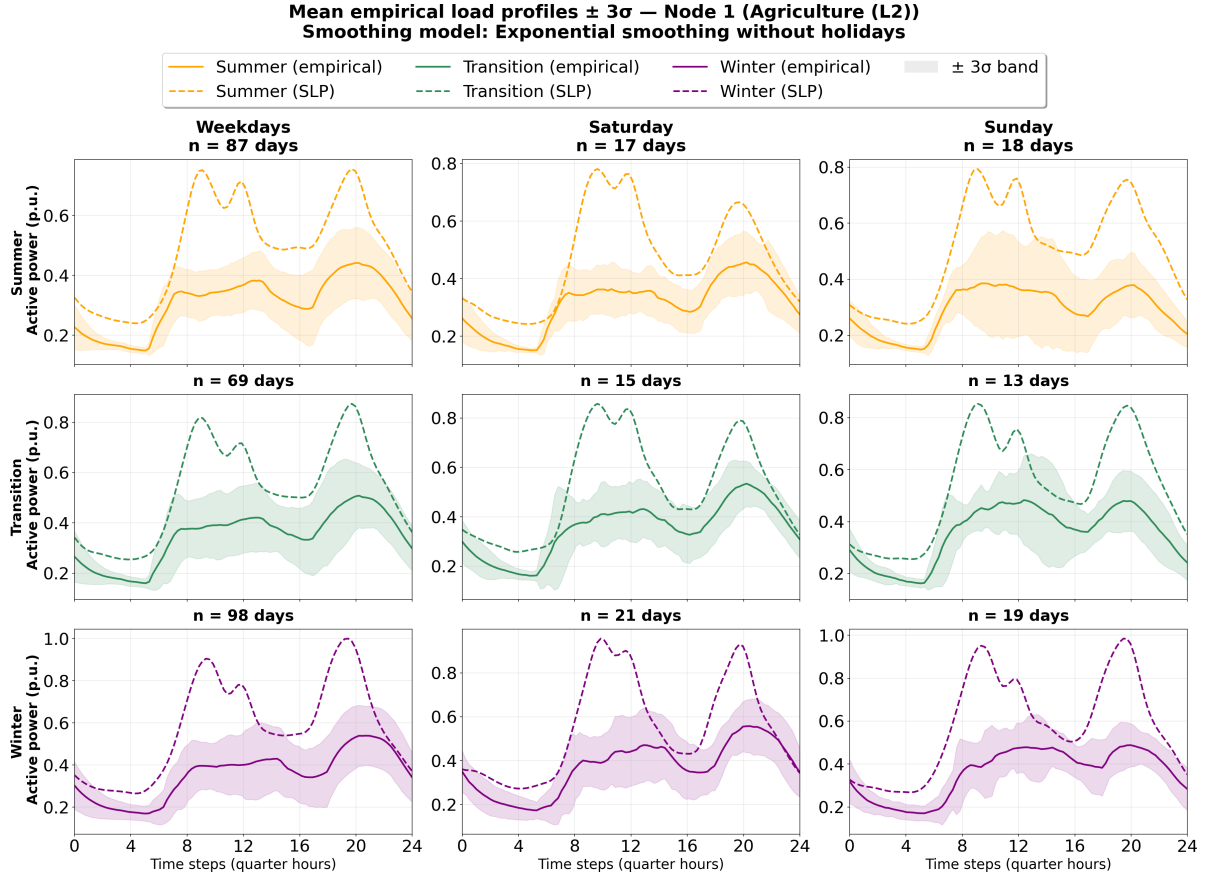


Figure 14: Mean empirical load profiles  $\pm 3\sigma$  for Node 1 (Agriculture, L2) smoothed with Exponential smoothing (without holidays).

(empirical load profiles) and Appendix A.3 (Stage 2 smoothed active power time series with holidays) as references.

#### 4.1.2.4. Effect of holidays on mean empirical profiles

Following the structure of Section 4.1.1.3, this section describes the impact of public holidays on the dataset. The focus is on comparing the empirical mean profiles against the mean SLPs. For reference, the corresponding plots that include public holidays are provided in Appendix A.3.

The mean profile curves and  $\pm 3\sigma$  bands are practically identical across all nodes, day types, seasons, and smoothing methods for both smoothed datasets. For Node 1 summer weekdays, the with-holidays and without-holidays smoothed datasets contain 88 and 87 days, respectively. Hence, excluding a single day from a group of 88 does not change the sample mean or standard deviation in meaningful way. The largest reduction occurs in the transition weekday group. Here, four holidays are removed (Good Friday, Easter Monday, Ascension Day, and Unity Day). This brings the count from 73 down to 69 days. Even this roughly 5 percent reduction does not produce any visible difference in

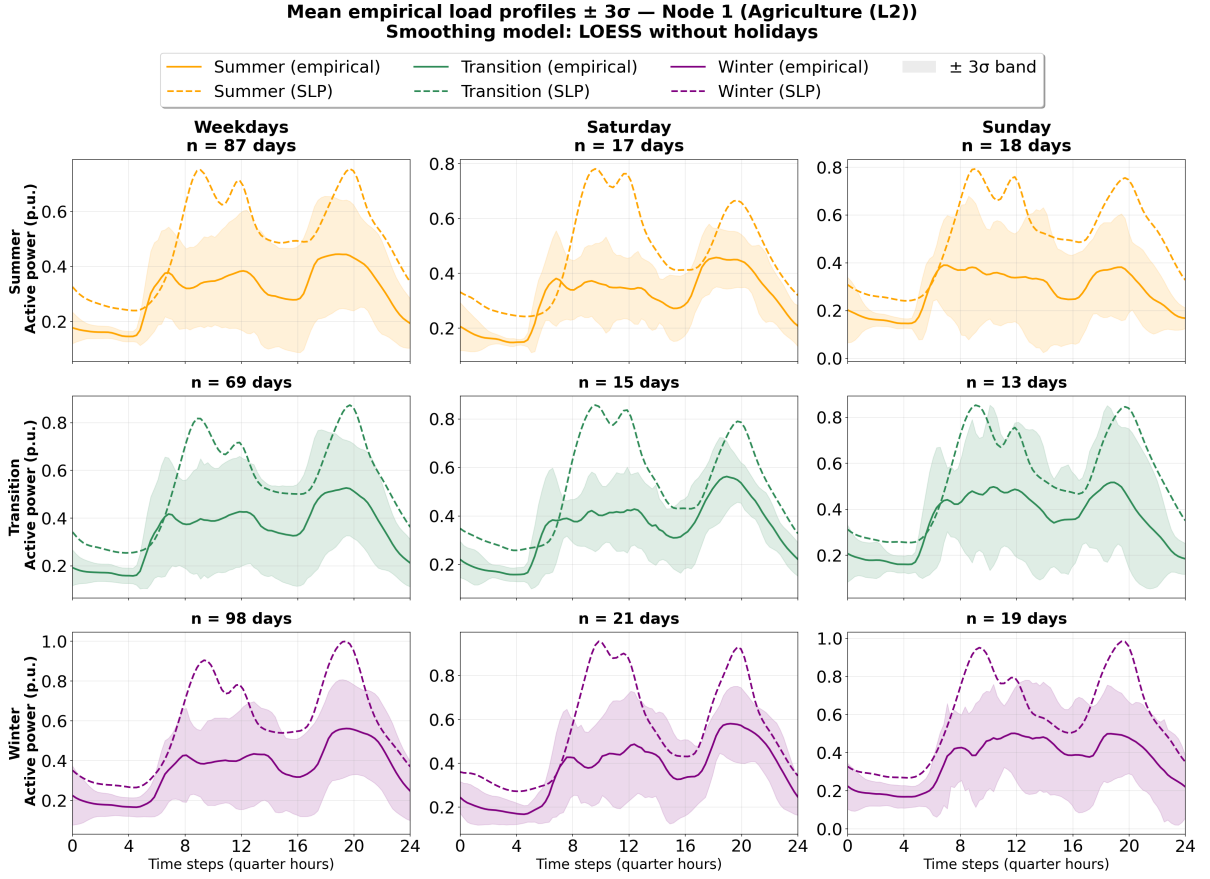


Figure 15: Mean empirical load profiles  $\pm 3\sigma$  for Node 1 (Agriculture, L2) smoothed with LOESS (without holidays).

the smoothed profiles or their uncertainty bands. The same conclusion holds for Node 8 and other nodes.

This result supports the theoretical argument made in the smoothing pipeline summary. Removing 1 to 4 days from a sample of 52 to 100 observations has a small effect on unusual consumption patterns. The mean profile is therefore robust to such minor changes in sample composition. Thus, holiday contamination does not measurably distort the empirical mean profiles computed in this study. Consequently, the dataset that excludes holidays is the appropriate choice for all subsequent analyses. Holidays introduce irregular patterns that do not reflect typical consumption behavior. To simplify referencing, the smoothed version of this dataset will be called the empirical profiles. Therefore, the empirical load profiles applies to clustering and MANOVA steps.

#### 4.1.2.5. Similarity of mean empirical profiles to Standard Load Profiles

This section shows that the empirical load profiles observed at individual nodes in the SimBench network deviate significantly from the BDEW SLPs for both consumer types. The SLP consistently overestimates per-unit consumption to the empirical mean. As in

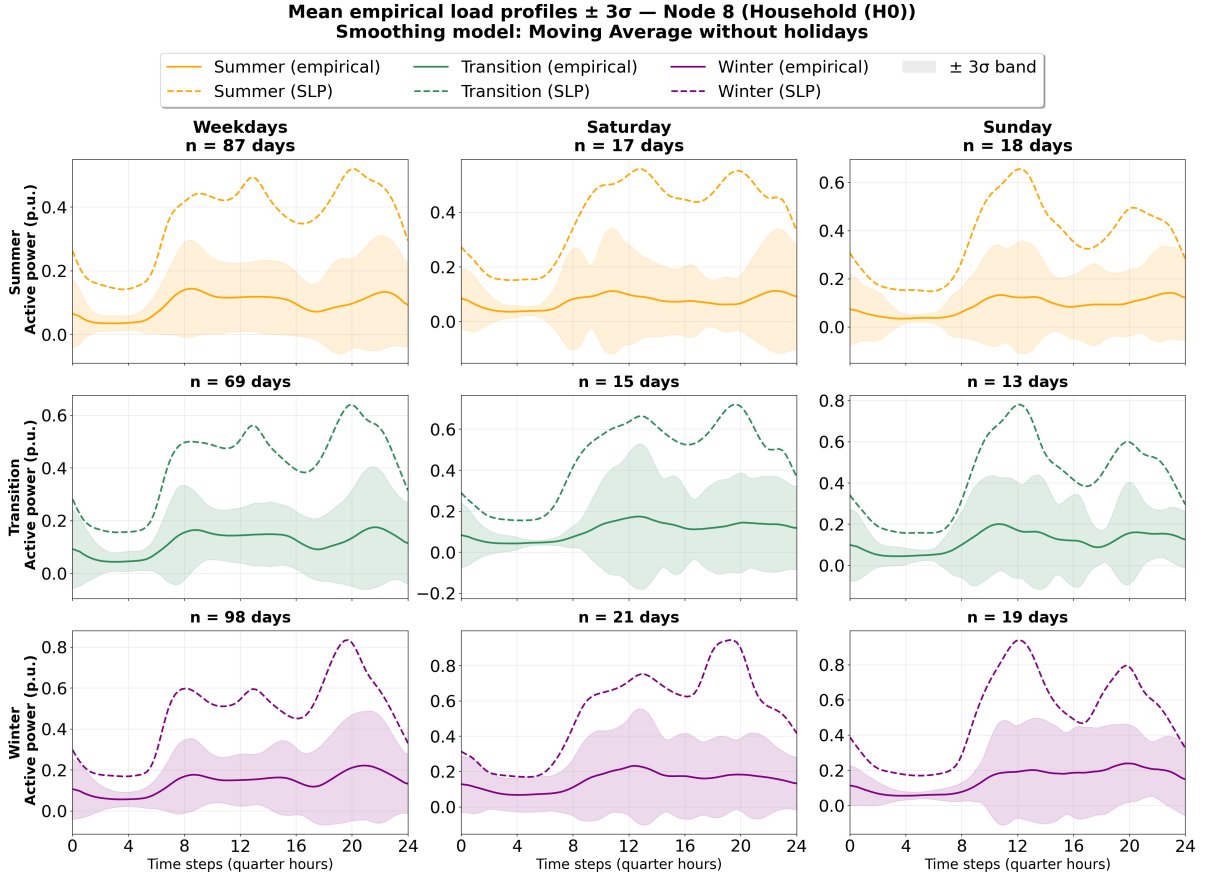


Figure 16: Mean empirical load profiles  $\pm 3\sigma$  for Node 8 (Household, H0) smoothed with Moving Average (without holidays).

most panels, the SLP lies above the upper  $\pm 3\sigma$  boundary of the observed daily profile distribution.

The reason for this mismatch is a difference in scale. The BDEW SLP is calculated by averaging normalized consumption across a large number of real consumers. This process may remove individual variation and generate a smoother structure. A single SimBench node does not benefit from this averaging effect. Its behavior is determined by only a few consumers, which makes it far more sensitive to individual variation. This is why household nodes tend to show nearly flat empirical means. The morning and evening peaks that are characteristic of the H0 profile do not appear at this level of aggregation. Agricultural nodes show a slightly different story. Their load shape is determined by the specific operational pattern of the synthetic load assigned to that node. This pattern is unique to each node and may look quite different from the broader L1 or L2 standard pattern that the SLP represents.

In addition, the per-unit normalization uses each node's annual peak as its reference point. Since this peak is driven by only a few extreme days in the year, the mean profiles within any seasonal group will naturally sit well below 1.0 p.u. The SLP is also normalized to its own annual peak. However, the SLP still shows a noticeably high magnitude

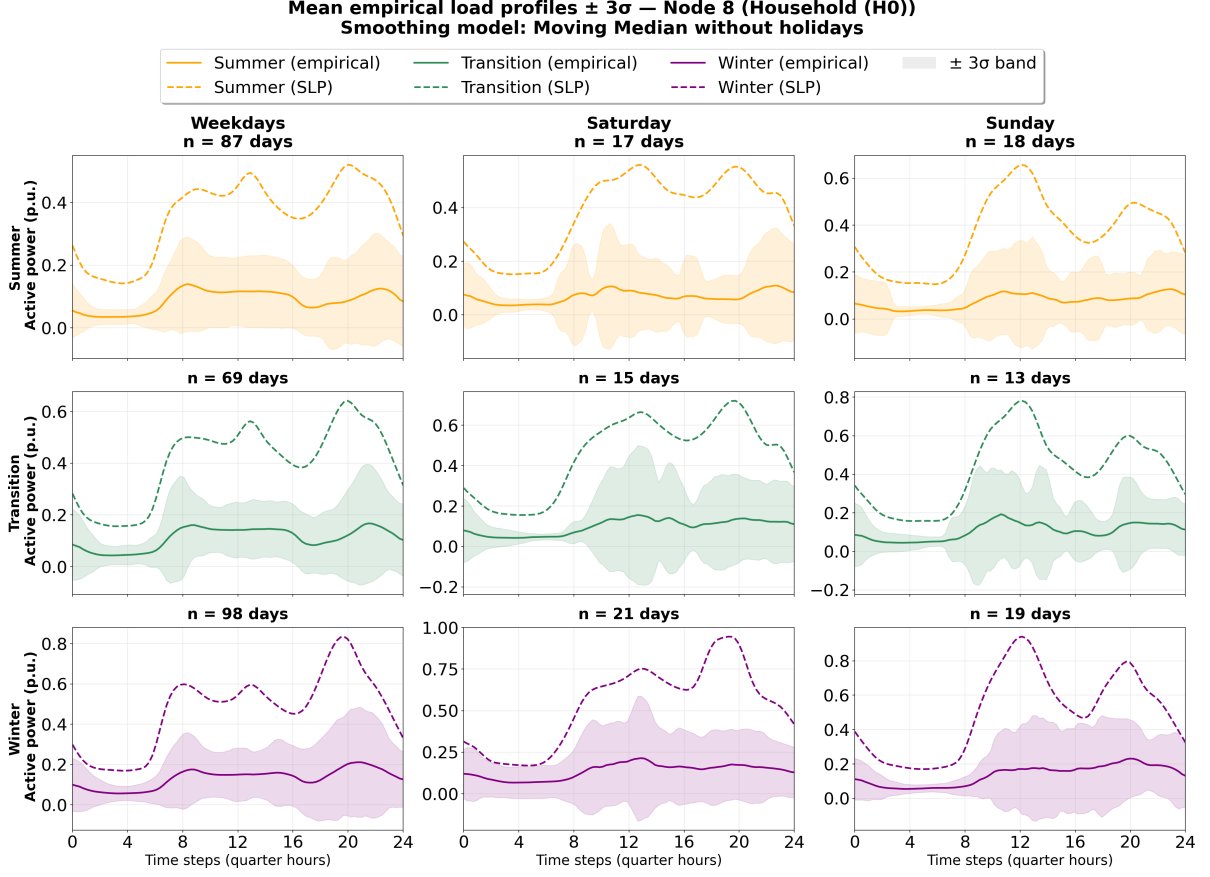


Figure 17: Mean empirical load profiles  $\pm 3\sigma$  for Node 8 (Household, H0) smoothed with Moving Median (without holidays).

during peak hours. This pattern holds even when the data is broken down into seasonal groups. This difference in behavior pushes the two profiles further apart. As a result, the gap between them appears wider than it might otherwise. The wide  $\pm 3\sigma$  bands at household nodes indicate strong day-to-day variability in load behavior. Agricultural nodes show a similar pattern though the bands are noticeably narrower. Together, these bands suggest that individual days at both node types do not consistently follow the shape of the corresponding SLP.

All smoothing methods show varying results. Moving Average and Moving Median produce the most stable results. Both methods produce smooth mean profiles. The uncertainty bands also remain stable across all node types. Exponential smoothing shows reasonable results. However, it tends to slightly raise the baseline during low consumption intervals. This can introduce a subtle distortion when comparing the smoothed profile against the SLP. LOESS is the most problematic method. The problems are obviously seen at household nodes and at Node 12. The uncertainty bands from LOESS are wider than those of the other methods. Their shape is also irregular. This behavior indicates that LOESS is less stable than the other methods in this context. The results here sup-

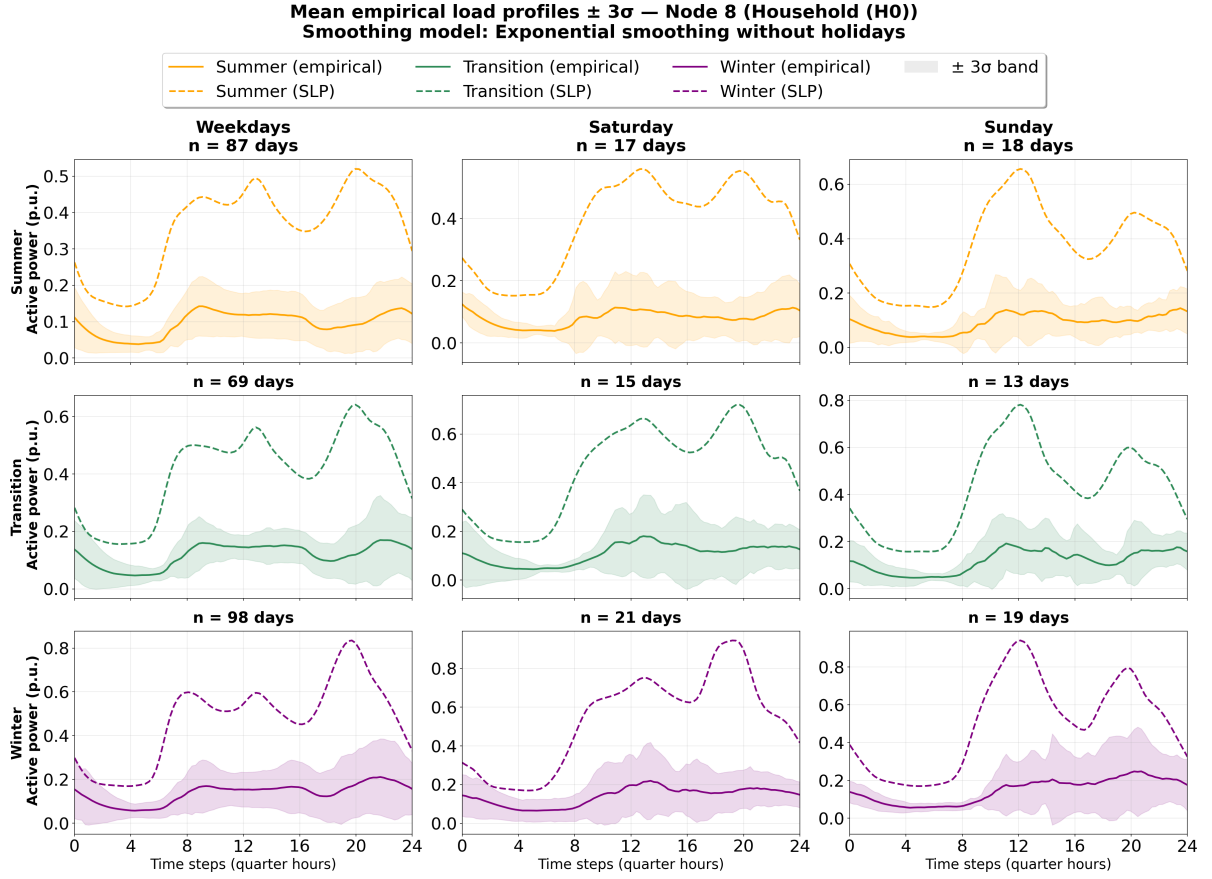


Figure 18: Mean empirical load profiles  $\pm 3\sigma$  for Node 8 (Household, H0) smoothed with Exponential smoothing (without holidays).

port the analysis in Section 4.1.1.3. Both the Moving Average and the Moving Median are more suitable for deriving active power time series in this case.

## 4.2. K-means clustering

This section describes the results of K-means clustering applied to the empirical load profiles across the five selected nodes. For each node, the daily profiles are grouped into three clusters. These clusters correspond to the three BDEW seasons. Clustering is performed separately for each day type and independently for each of the four smoothing methods. Three distance measures are conducted in this study. These are Euclidean distance, Dynamic Time Warping (DTW), and correlation-based distance.

Two metrics are used to measure the quality of the resulting clusters. These are Silhouette score and Correct Classification Rate (CCR). The Silhouette score evaluates the quality of the clusters in two ways. It measures how compact each cluster is and how well it is separated from the others. A score closer to 1.0 suggests that the clusters are clearly distinct from each other. The CCR measures how well the obtained clusters align with the known seasonal labels. Since the assignment of cluster indices to season categories is not

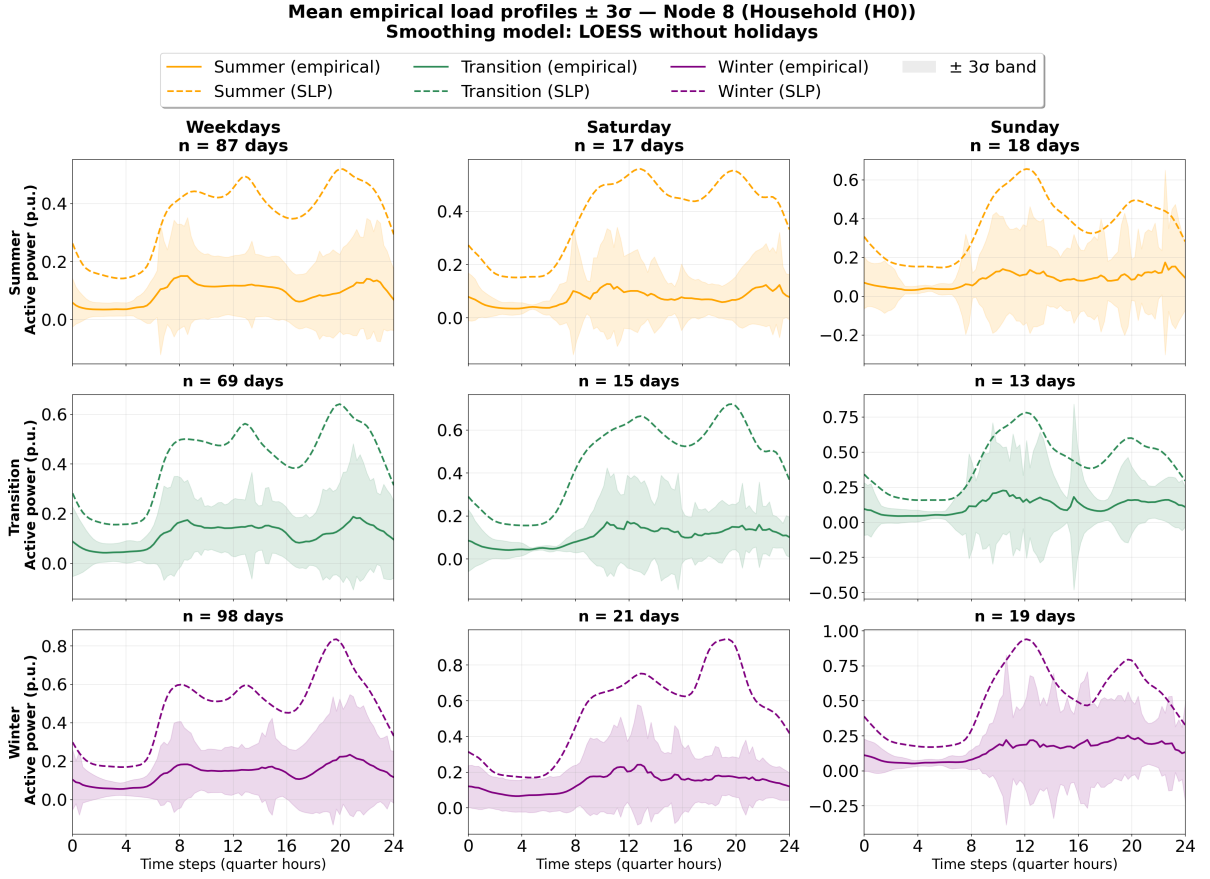


Figure 19: Mean empirical load profiles  $\pm 3\sigma$  for Node 8 (Household, H0) smoothed with LOESS (without holidays).

fixed in advance, all six possible permutations of the three cluster to season assignments are evaluated. The clusters with the highest proportion of correctly classified days are selected. Both measurements assess whether the power consumption patterns at each node align with the seasonal structure prescribed by the BDEW framework.

The results are visualized using bar plots. Each plot corresponds to a specific node and distance measure. The four smoothing methods are displayed along the horizontal axis. Each plot contains three panels. Every panel represents one of the three day types. The orange dashed horizontal line at 0.50 marks the reference threshold for both metrics. The same representative nodes from Section 4.1 are discussed here. These are Nodes 1 (agriculture) and 8 (household). The results for the remaining nodes are discussed separately. In addition, the corresponding figures are provided in Appendix A.4.

#### 4.2.1. Empirical load profiles clustering of agriculture representative node: Node 1

The clustering result for Node 1 is consistent with the empirical mean profiles presented in Section 4.1.2.1. It reveals clear seasonal differences in consumption levels. Winter profiles are higher than both summer and transition period means. However, the profile shape

itself does not change dramatically between seasons. The degree of similarity between profiles affects the performance of distance metrics. The Node 1 clustering results under all three distance metrics are shown in Figures 20, 21, and 22.

### Silhouette score and Correct Classification Rate (CCR): K-Means clustering with Correlation-based distance — Node 1

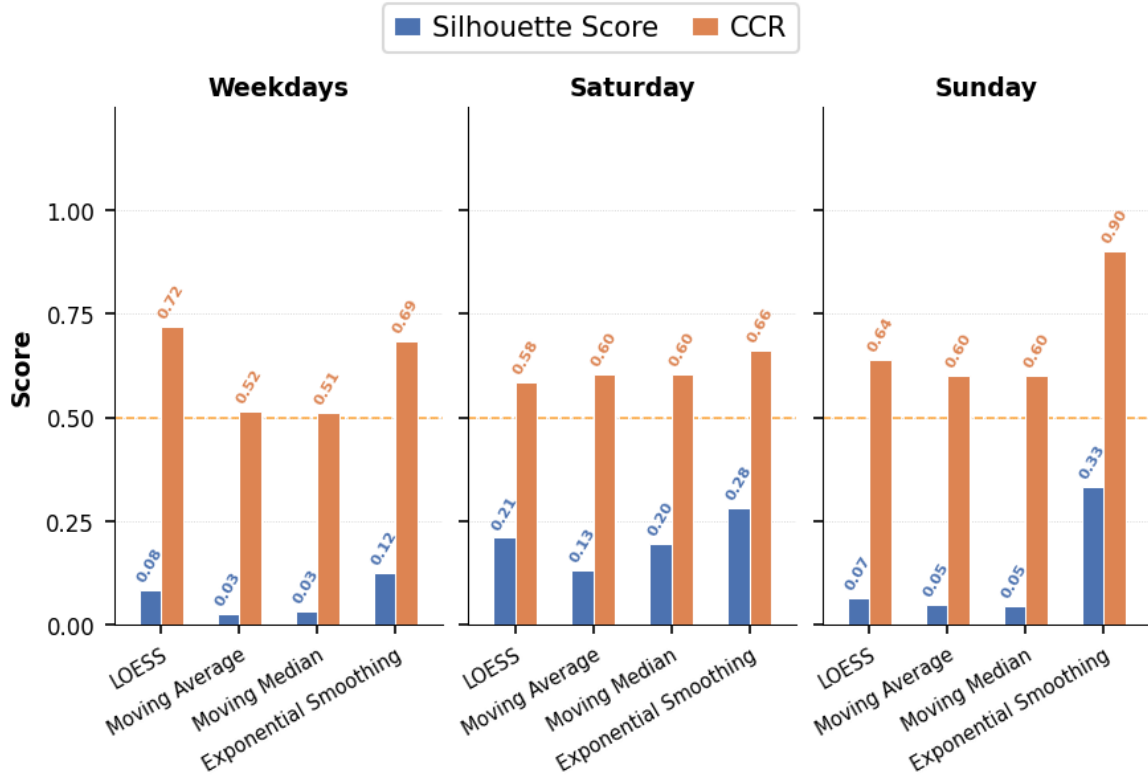


Figure 20: Silhouette score and Correct Classification Rate (CCR) for K-Means clustering with Correlation-based distance for Node 1.

Silhouette scores for Node 1 are consistently low across all three distance metrics. Under the correlation-based metric, scores range from approximately 0.03 to 0.33, with Exponential smoothing reaching the highest value on Saturdays. The DTW scores are below the 0.50 threshold, with a range from 0.19 to 0.35. Scores fall between 0.19 and 0.36 for Euclidean distance. The low Silhouette values reflect the partial overlap visible in the seasonal  $\pm 3\sigma$  bands in Figures 12 to 15, where summer, transition, and winter distributions do not occupy clearly separate regions of the profile space.

Despite the low Silhouette scores, CCR values for Node 1 are considerably more encouraging for all distance metrics. CCR ranges from approximately 0.51 to 0.90, 0.47 to 0.72, and 0.51 to 0.90 for correlation-based distance, DTW, and Euclidean distance, respectively. Saturday shows the highest CCR in the correlation-based distance, with a CCR of 0.90 for Exponential smoothing. Under DTW, Sunday panels perform the best, with Moving



### Silhouette score and Correct Classification Rate (CCR): K-Means clustering with DTW — Node 1

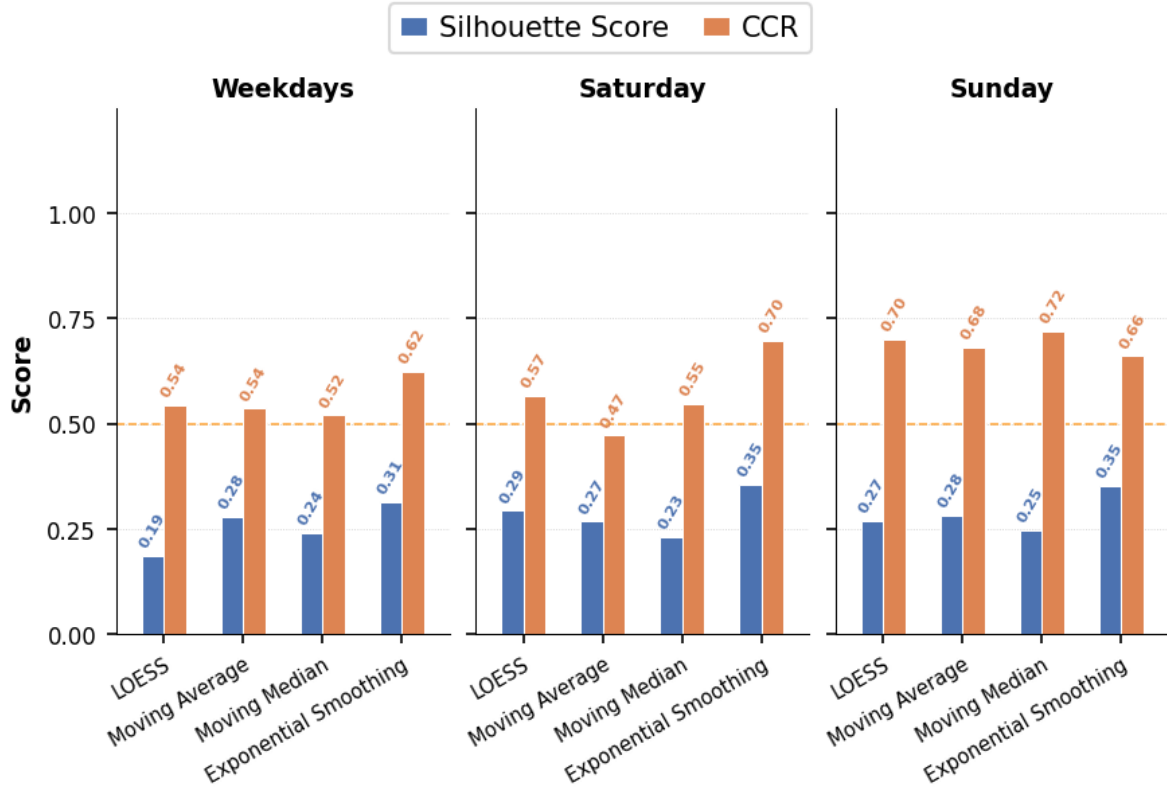


Figure 21: Silhouette score and Correct Classification Rate (CCR) for K-Means clustering with DTW distance for Node 1.

Median reaching 0.72. The correlation-based metric on weekdays slightly outperforms DTW.

Euclidean distance produces the widest CCR range for Node 1, with values approximately 0.51 to 0.90. The most notable result is Exponential smoothing on Sunday, with the CCR values at 0.90. This is the highest single clustering result observed for this node across all distance metrics. The LOESS also achieves a high CCR of 0.72 on weekdays. These results suggest that the Euclidean distance is well suited to recovering seasonal structure, particularly when combined with Exponential smoothing for an agricultural node.

Overall, Exponential smoothing consistently produces the highest Silhouette scores and CCR values across all three distance metrics and all three day types at Node 1. LOESS and Moving Average perform comparably on weekdays. Moving Median tracks closely with Moving Average, which is consistent with the findings from Section 4.1. This pattern indicates that noise within a day and across weeks affects seasonal clusters. Exponential smoothing tends to produce profiles with slightly more distinct seasonal patterns.



### Silhouette score and Correct Classification Rate (CCR): K-Means clustering with Euclidean distance — Node 1

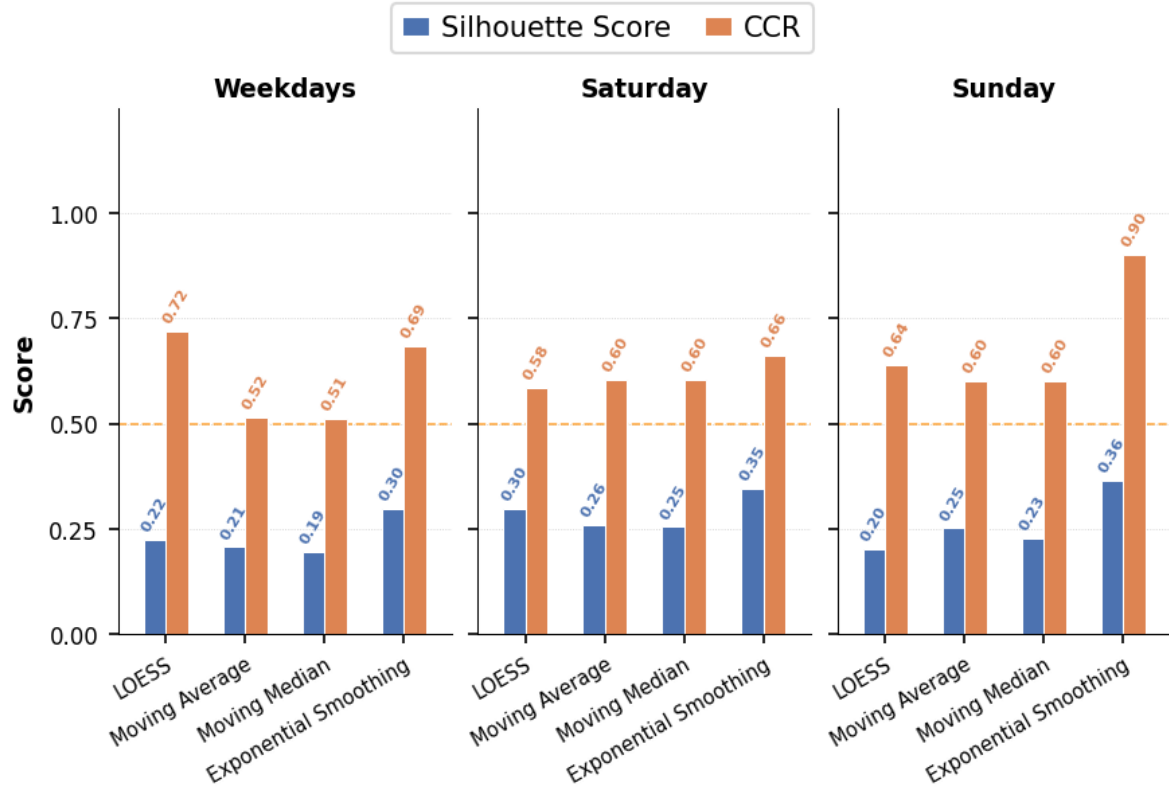


Figure 22: Silhouette score and Correct Classification Rate (CCR) for K-Means clustering with Euclidean distance for Node 1.

#### 4.2.2. Empirical load profiles clustering of household representative node: Node 8

Node 8 shows a weaker clustering picture than Node 1. This is consistent with the nearly flat profiles observed at Node 8 (Section 4.1.2.2). There is almost no structured variation within the day. In addition, the mean consumption level changes very little between seasons. Since the profiles show very little variation, the clustering algorithm struggles to find clear boundaries between groups. The clustering results for Node 8 are displayed in Figures 23, 24, and 25.

Node 8 has the lowest Silhouette scores among the five representative nodes in this study. No scores across all distance metrics and smoothing methods achieve a threshold above 0.50. The correlation-based metric shows a Silhouette value between  $-0.6$  and  $0.13$ . For DTW, the range is wider than the correlation-based distance, with the scores from  $0.15$  to  $0.46$ . For the Euclidean distance, the values range from  $0.16$  to  $0.32$ . The low Silhouette score shows that Node 8 does not have clear seasonal patterns, consistent with the mean load profile analysis

### Silhouette score and Correct Classification Rate (CCR): K-Means clustering with Correlation-based distance — Node 8

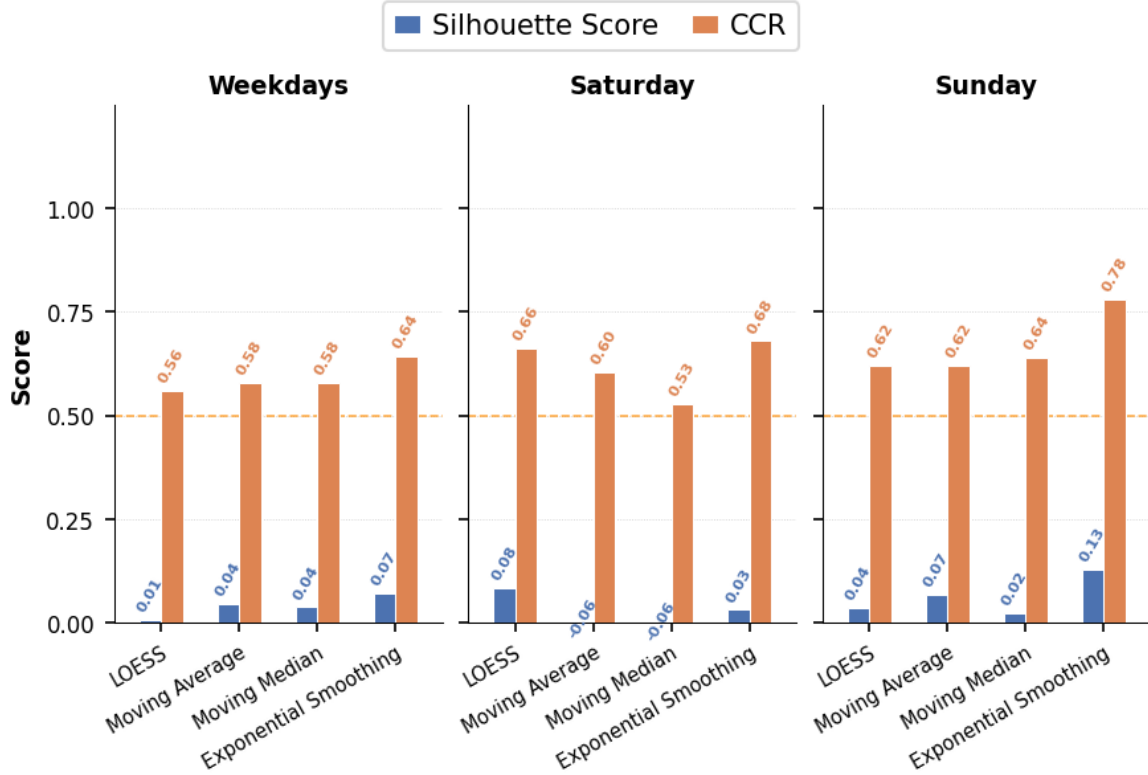


Figure 23: Silhouette score and Correct Classification Rate (CCR) for K-Means clustering with Correlation-based distance for Node 8.

The CCR values for Node 8 under correlation-based distance are above the 0.50 threshold in all configurations. The weekday CCR values are between 0.56 and 0.64 for the correlation-based distance. Performance is slightly better on Saturday and Sunday. Exponential smoothing gives a CCR of 0.68 for Saturdays and 0.78 for Sundays. These CCR values show that the correlation metric can capture the shape of the daily profile even the profile shapes of Node 8 are nearly identical across seasons.

DTW gives slightly better CCR values than the correlation metric. On weekdays, the values range from 0.52 to 0.58, where CCR on Saturdays and Sundays reaches 0.62. However, the improvement over the correlation metric is still small, and Silhouette scores remain low. This shows that DTW's flexibility offers little benefit when the daily profile varies.

K-means clustering with Euclidean distance gives the highest CCR values. The CCR reaches 0.78 when Exponential smoothing is used. On weekdays, CCR values are between 0.56 and 0.64 for Euclidean distance. Saturday values reach up to 0.68, and Sunday values reach up to 0.78 under Exponential smoothing. Winter profiles tend to lie slightly higher than summer ones. Euclidean distance is sensitive to these absolute level

### Silhouette score and Correct Classification Rate (CCR): K-Means clustering with DTW — Node 8

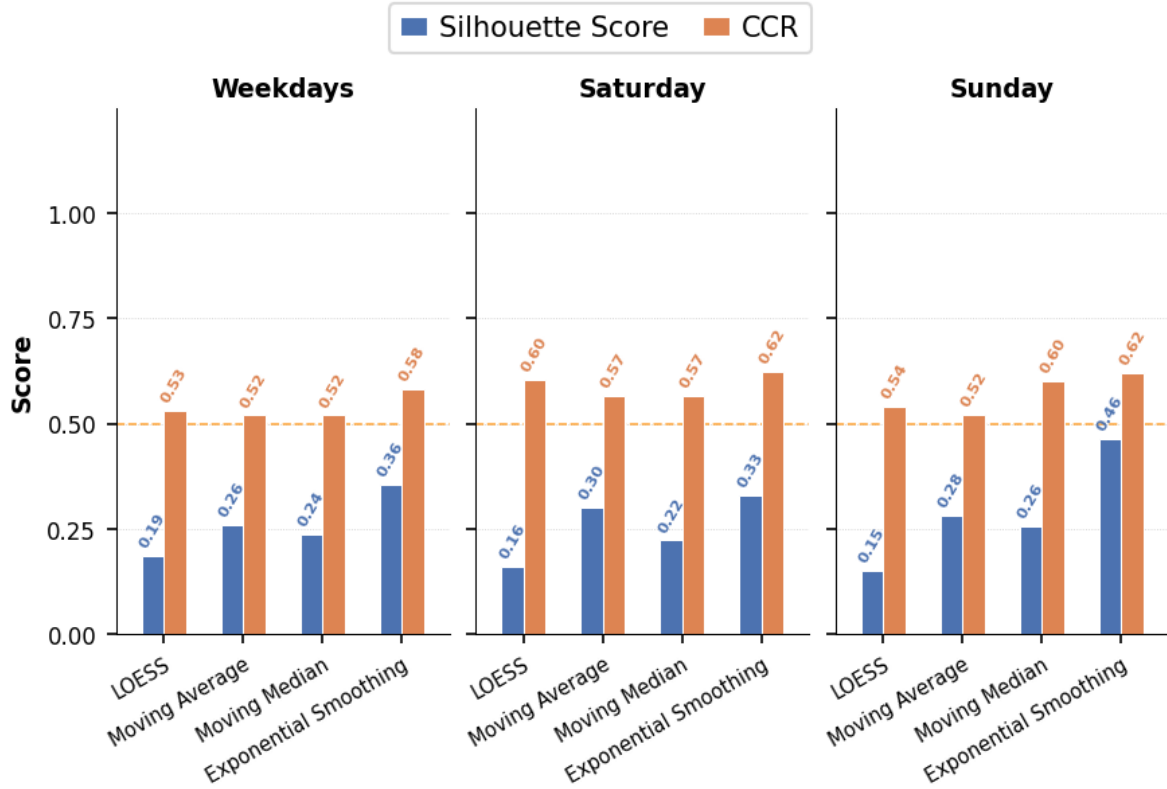


Figure 24: Silhouette score and Correct Classification Rate (CCR) for K-Means clustering with DTW distance for Node 8.

differences. While correlation-based distance and DTW are more sensitive to shape and timing patterns. These two metrics struggle to show meaningful differences. Hence, the Euclidean distance can detect a small change in winter consumption levels. This allows it to partially separate winter profiles into their correct seasonal cluster.

Exponential smoothing once again stands out as the best performing smoothing method at this node. It shows the highest Silhouette scores and CCR values across all three distance metrics. Moving Average and Moving Median deliver very similar results. On the other hand, LOESS tends to give the lowest CCR on weekdays. One possible reason Exponential smoothing performs better here is its ability to retain a decaying memory of past consumption values. This property slightly amplifies small seasonal differences, enabling partial cluster separation

#### 4.2.3. Empirical load profiles clustering of remaining nodes

This section briefly discusses the clustering results for Nodes 11, 12, and 13. The clustering charts for these nodes are provided in Appendix A.4. Node 12 is an agricultural consumer

### Silhouette score and Correct Classification Rate (CCR): K-Means clustering with Euclidean distance — Node 8

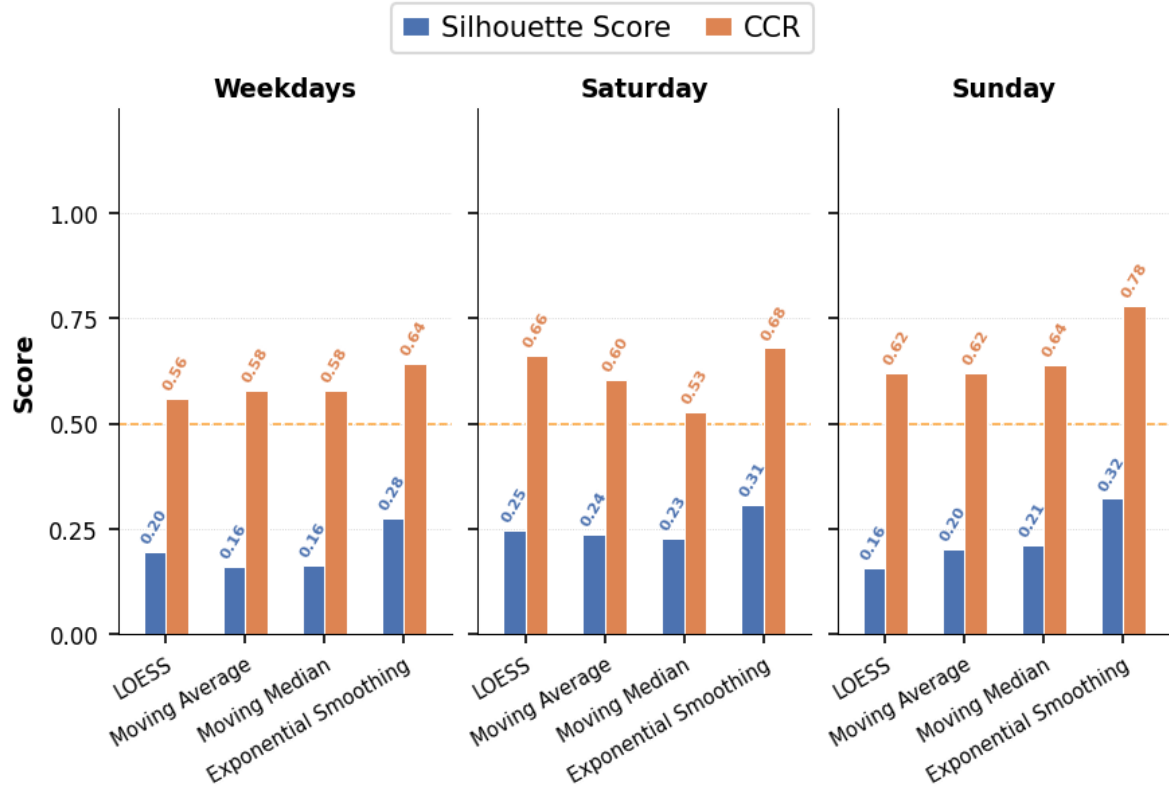


Figure 25: Silhouette score and Correct Classification Rate (CCR) for K-Means clustering with Euclidean distance for Node 8.

assigned the L1 standard load profile, while Nodes 11 and 13 are both household consumers assigned the H0 profile.

Node 11 shows strong clustering performance among all five representative nodes. It achieves better results under DTW and Euclidean distance than the other four nodes. The Silhouette scores range of DTW is from 0.46 to 0.62. All values meet the 0.50 threshold. The only exception is LOESS, which is 0.46 and 0.48 on weekdays and on Sunday, respectively. With DTW, the Moving Average and Moving Median methods achieve CCR values of 0.79 on weekdays and 0.86 on Sundays. The Silhouette scores are between 0.43 and 0.58 for Euclidean distance. CCR values for this metric range from 0.66 to 0.80 across day types. The correlation-based metric also shows strong performance. With Exponential smoothing, its CCR reaches 0.80 on Sundays. The superior performance of Node 11 relates to its mean load profile shown in Appendix A.2. A clear evening peak is visible in the winter weekday and Sunday panels. This additional intraday structure allows DTW and Euclidean distance to capture seasonal shape differences. Whereas Nodes 8 and 13 do not exhibit this pattern. Despite other smoothing methods, Exponential smoothing shows a CCR of only 0.52 for Node 11 on Sundays when DTW is applied.

Node 12 presents moderate clustering performance. DTW shows strong weekday Silhouette scores. The Moving Average shows the highest score of 0.65 across all nodes. However, the CCR of DTW on weekdays is only between 0.49 and 0.58. The high Silhouette scores suggest that the three seasonal clusters formed with DTW are internally coherent. However, the moderate CCR indicates that these clusters do not align well with the actual summer, transition, and winter labels. As shown in the mean load profile plots in Appendix A.2, Node 12 exhibits two narrow peaks occurring in the morning and evening. This can lead DTW to form clusters driven by shape similarity rather than seasonal behavior. Correlation-based distance achieves comparable CCR to DTW for Node 12 on weekdays and Saturdays, with Moving Median reaching 0.55 on weekdays and 0.57 on Saturdays. Euclidean distance achieves moderate scores, with CCR in the range of 0.50 to 0.60.

Node 13 consistently shows poor clustering results across all distance metrics and all smoothing methods. CCR values are mostly below 0.50. However, Exponential smoothing under the correlation-based metric shows the CCR at 0.58. Silhouette scores rarely exceed 0.38. The most extreme result is the DTW Silhouette score of 0.09 for LOESS on Sundays. This performance pattern is directly consistent with the behavior of Node 13 described in Section 4.1. It suggests Node 13 shows the highest variability among the five nodes. Especially, LOESS struggled to capture its profile shape reliably. This high variability makes it difficult for any distance metric to form stable seasonal clusters.

#### **4.2.4. Empirical load profile patterns in clustering performance**

The clustering results show consistent patterns. However, Silhouette scores are low in most cases, rarely above 0.50 across all metrics and smoothing methods. This suggests that seasonal clusters are poorly separated in the active power time series. Furthermore, the uncertainty bands show overlap at every node. Therefore, the mean load profile analysis supports the Silhouette score results.

CCR values are higher than the Silhouette scores. It gives a better view of seasonal recovery. Agricultural nodes, especially Node 1, show strong CCR values under Euclidean and correlation-based distances when the appropriate smoothing method is used. Household nodes clearly depend on the choice of distance metric. At Node 13, Euclidean distance works better because the seasonal difference is only in the consumption level, not in the profile shape. Euclidean and correlation-based distances perform comparably at Node 8. Both capture the small but consistent difference in winter consumption level. Node 11 stands out among household nodes. It performs well under DTW, Euclidean, and correlation-based distances because its winter profiles have a small daily pattern.

Exponential smoothing performs best in most cases in most node and day type combinations, regardless of the distance metric used. Moving Average and Moving Median deliver similar, though slightly lower. LOESS shows unstable performance. It works well at stable nodes like Node 1 but performs poorly at nodes with high variability like Node 13. As discussed in Section 4.1, Moving Average and Moving Median are identified as the most consistent smoothing methods.

### 4.3. MANOVA tests on empirical and residual profiles

This section explains the reason for the visual evidence discussed in Section 4.1 and the cluster quality metrics from Section 4.2. A significance level of  $\alpha_{sig} = 0.05$  is applied to all tests.

Before interpreting the MANOVA results, the four standard assumptions of the test are considered. The first assumption is satisfied, as the smoothed daily active power time series are continuous by nature. For the second assumption, multivariate normality of the residuals is not strictly met due to the characteristics of the data. However, by the Multivariate Central Limit Theorem, the distribution of the sample mean vector approaches multivariate normality as the sample size increases (Tabachnick and Fidell, 2014, p. 294). Given the relatively large number of daily observations in this study, this assumption is considered acceptable. The variance-covariance matrices across groups may differ in practice. Nevertheless, they are assumed to be equal for the purpose of this analysis. The daily profiles are derived from a continuous yearly time series and exhibit temporal dependencies. Thus, the independence of observations cannot be guaranteed. This assumption is likewise neglected. This limitation is acknowledged. Overall, while not all assumptions are perfectly satisfied. The MANOVA is still applied, and the results are interpreted accordingly.

Nodes 1 and 8 again serve as the representative nodes and are discussed in detail. The results of Nodes 11, 12, and 13 are also briefly discussed here.

#### 4.3.1. MANOVA Test A: Season, day type, and interaction effects on empirical load profiles

The MANOVA results for Test A are presented in Tables 4 to 7. Each table reports the Pillai's trace  $p$ -values for the season, day type, and interaction effects. Their interaction for each node under a single smoothing method.

Table 4: MANOVA Test A for Moving Average.

| Node                  | Season                 | Day Type               | Interaction           |
|-----------------------|------------------------|------------------------|-----------------------|
| 1 (Agricultural, L2)  | $2.45 \times 10^{-15}$ | $6.56 \times 10^{-5}$  | $1.29 \times 10^{-3}$ |
| 8 (Household, H0)     | $4.51 \times 10^{-9}$  | $8.49 \times 10^{-7}$  | $4.31 \times 10^{-5}$ |
| 11 (Household, H0)    | $1.04 \times 10^{-9}$  | $> 0.999$              | 0.052                 |
| 12 (Agricultural, L1) | $1.81 \times 10^{-14}$ | $6.17 \times 10^{-13}$ | $1.78 \times 10^{-8}$ |
| 13 (Household, H0)    | 0.049                  | 0.020                  | 0.0083                |

Table 5: MANOVA Test A for Moving Median.

| Node                  | Season                 | Day Type               | Interaction            |
|-----------------------|------------------------|------------------------|------------------------|
| 1 (Agricultural, L2)  | $1.63 \times 10^{-9}$  | $1.82 \times 10^{-3}$  | 0.042                  |
| 8 (Household, H0)     | $9.90 \times 10^{-14}$ | $4.70 \times 10^{-4}$  | $1.35 \times 10^{-9}$  |
| 11 (Household, H0)    | $2.27 \times 10^{-12}$ | $> 0.999$              | $6.03 \times 10^{-5}$  |
| 12 (Agricultural, L1) | $2.02 \times 10^{-18}$ | $3.42 \times 10^{-20}$ | $1.82 \times 10^{-15}$ |
| 13 (Household, H0)    | 0.031                  | 0.0037                 | 0.023                  |

#### 4.3.1.1. Hypotheses: MANOVA Test A

The three sets of hypotheses tested with  $\alpha_{sig} = 0.05$  for this test are defined as in Section 3.3.5.

Season effect:

$$H_0^{(S)} : \bar{\mu}_{s_1} = \bar{\mu}_{s_2} = \bar{\mu}_{s_3}$$

$$H_1^{(S)} : \text{not all } \bar{\mu}_{s_1} = \bar{\mu}_{s_2} = \bar{\mu}_{s_3}$$

Day type effect:

$$H_0^{(\delta)} : \bar{\mu}_{\delta_1} = \bar{\mu}_{\delta_2} = \bar{\mu}_{\delta_3}$$

$$H_1^{(\delta)} : \text{not all } \bar{\mu}_{\delta_1} = \bar{\mu}_{\delta_2} = \bar{\mu}_{\delta_3}$$

Season  $\times$  day type interaction:

$$H_0^{(S \times \delta)} : \mu_{s,\delta} - \bar{\mu}_s - \bar{\mu}_\delta + \bar{\mu} = \mathbf{0} \quad \forall s, \delta$$

$$H_1^{(S \times \delta)} : \exists (s, \delta) \text{ such that } \mu_{s,\delta} - \bar{\mu}_s - \bar{\mu}_\delta + \bar{\mu} \neq \mathbf{0}$$

where each hypothesis is evaluated independently and the corresponding  $p$ -values are reported for each node and smoothing method in Tables 4 to 7.

#### 4.3.1.2. MANOVA Test A of agriculture representative node: Node 1

Node 1 shows significant seasonal effects with all smoothing methods. The season effect  $p$ -

Table 6: MANOVA Test A for Exponential smoothing.

| Node                  | Season                 | Day Type               | Interaction             |
|-----------------------|------------------------|------------------------|-------------------------|
| 1 (Agricultural, L2)  | $2.43 \times 10^{-63}$ | $5.46 \times 10^{-81}$ | $5.01 \times 10^{-88}$  |
| 8 (Household, H0)     | $9.27 \times 10^{-65}$ | $9.66 \times 10^{-80}$ | $4.23 \times 10^{-101}$ |
| 11 (Household, H0)    | $3.64 \times 10^{-45}$ | $1.08 \times 10^{-9}$  | $2.07 \times 10^{-67}$  |
| 12 (Agricultural, L1) | $2.34 \times 10^{-70}$ | $1.99 \times 10^{-84}$ | $3.83 \times 10^{-103}$ |
| 13 (Household, H0)    | $3.96 \times 10^{-46}$ | $2.47 \times 10^{-64}$ | $3.17 \times 10^{-87}$  |

Table 7: MANOVA Test A for LOESS.

| Node                  | Season                 | Day Type               | Interaction            |
|-----------------------|------------------------|------------------------|------------------------|
| 1 (Agricultural, L2)  | $3.07 \times 10^{-28}$ | $2.27 \times 10^{-10}$ | $3.58 \times 10^{-20}$ |
| 8 (Household, H0)     | $1.71 \times 10^{-29}$ | $9.56 \times 10^{-19}$ | $1.07 \times 10^{-40}$ |
| 11 (Household, H0)    | $3.92 \times 10^{-26}$ | $> 0.999$              | $1.22 \times 10^{-15}$ |
| 12 (Agricultural, L1) | $1.46 \times 10^{-40}$ | $1.16 \times 10^{-39}$ | $2.27 \times 10^{-43}$ |
| 13 (Household, H0)    | $1.01 \times 10^{-9}$  | $5.55 \times 10^{-11}$ | $5.60 \times 10^{-19}$ |

value is  $2.45 \times 10^{-15}$  for Moving Average and  $1.63 \times 10^{-9}$  for Moving Median, respectively. These results show that the mean empirical load profiles for Node 1 vary across the three seasons. This matches what is seen in Figures 12 to 15, where winter profiles have higher active power levels than summer and transition profiles, especially in the evening. The MANOVA confirms what the visual analysis suggested.

The day type effect is also significant under all four smoothing methods. The  $p$ -values range from  $1.82 \times 10^{-3}$  under Moving Median to  $6.56 \times 10^{-5}$  under Moving Average. The mean profile plots in Section 4.1.2.1 showed similar load shapes on weekdays, Saturdays, and Sundays for Node 1. However, MANOVA can reveal small differences that may not be obvious when only the mean curves are considered. Furthermore, the weekday group contains far more observations than the weekend groups. This imbalance can also affect the test. The significant day type result is therefore plausible even when the visual profiles appear similar.

The interaction effect is significant in all methods. Under Moving Average, the  $p$ -value is  $1.29 \times 10^{-3}$ , and under Moving Median is 0.042. Which is marginally significant. A significant interaction indicates that the extent of seasonal variation differs across all day types. The weekend shows wider uncertainty bands and fewer observations per group in Section 4.1.2.1. This reflects less regular agricultural activity on Saturdays and Sundays. Therefore, the seasonal signal has a different magnitude on weekends than on weekdays.

The season  $p$ -value under Exponential smoothing is  $2.43 \times 10^{-63}$ , compared to  $2.45 \times 10^{-15}$  under Moving Average. This large discrepancy is not a sign of a stronger underlying effect. Rather, it reflects the known behavior of Exponential smoothing identified in Section 4.1.1.3, where the method introduces a systematic temporal lag and an elevated



baseline. These artifacts widen the apparent separation between seasonal group centroids and inflate the test statistics. The main conclusion is the same for all methods, but the Exponential smoothing  $p$ -values cannot be directly compared in size to those from the other three methods.

The MANOVA results for Node 1 match the clustering findings in Section 4.2.1. In that section, CCR values above 0.50 are observed for correlation-based and Euclidean distance across all day types, while DTW returns a minimum CCR of 0.47. This indicates that seasonal structure is at least partially evident in the active power time series across most distance metrics. Again, MANOVA gives formal statistical support for what the clustering already showed.

#### 4.3.1.3. MANOVA Test A of household representative node: Node 8

Node 8 shows statistically significant results for all three effects across all smoothing methods. For Moving Average, the season, day type, and interaction  $p$ -values are  $4.51 \times 10^{-10}$ ,  $8.49 \times 10^{-10}$ , and  $4.31 \times 10^{-5}$ , respectively. The same corresponding  $p$ -values are  $9.90 \times 10^{-14}$ ,  $4.70 \times 10^{-4}$ , and  $1.35 \times 10^{-9}$  under Moving Median. These results provide strong statistical evidence for all three effects.

These results seem to contradict the empirical mean profile analysis in Section 4.1.2.2. It describes the active power profiles at Node 8 as almost flat across all seasons and day types. There are also no clear morning or evening peaks. The MANOVA uses the full 96-dimensional profile vector for each day. Hence, it makes it sensitive to small but consistent differences.

The clustering results in Section 4.2.2 support this interpretation. For Node 8, the Silhouette scores are below 0.50 for all distance metrics and smoothing methods. CCR values based on correlation-based distance are above 0.50 across all day type. While DTW provides only slight improvements. Therefore, these analyses show that while MANOVA detects a seasonal signal at Node 8, the clustering results reveal that this signal is too weak and similar to create useful seasonal groups. A statistically significant MANOVA result does not mean the H0 standard load profile fits Node 8 well.

Similar to Node 1, Exponential smoothing gives much more extreme  $p$ -values. For example, the season  $p$ -value with Exponential smoothing is  $9.27 \times 10^{-65}$ . This happens for the same reason as before. Exponential smoothing raises the baseline. This slightly increases the small seasonal differences at Node 8. Hence, it makes the group separation appear larger in MANOVA than it is. Moving Average and Moving Median give consistent and similar results that match their nearly identical smoothing behavior described in Section 4.1.

#### 4.3.1.4. MANOVA Test A on remaining nodes

Season, day type, and interaction effects are highly significant for every smoothing method for Node 12. The season  $p$ -value is  $2.02 \times 10^{-18}$  with Moving Median and  $2.34 \times 10^{-70}$  with Exponential smoothing. Node 12 has a clear morning and evening peak, as described in Section 4.1.2.3. This daily pattern provides MANOVA with more information to separate seasonal groups, since both the shape and the level of the profiles change. The strong results across all smoothing methods indicate large seasonal differences at this node.

Node 11 (Household, H0) shows a clear difference in day type results. For Moving Average, Moving Median, and LOESS, the day type  $p$ -value is above 0.999. These indicate no effect. The day type  $p$ -value of Exponential smoothing is highly significant with  $1.08 \times 10^{-9}$ . This difference is due to how Exponential smoothing works, as explained in Section 4.1.1.3. Hence, it may create differences between day types. The three more neutral methods agree that there is no day type effect at Node 11. However, the season effect is strongly significant for all four methods, with  $p$ -values from  $1.04 \times 10^{-9}$  to  $3.64 \times 10^{-45}$ . This matches the winter seasonality seen in the mean empirical load profiles for Node 11 in Appendix A.2. The interaction effect is significant with Moving Median, Exponential smoothing, and LOESS, but just misses significance with Moving Average at the  $p$ -value of 0.052.

Node 13 (Household, H0) has the weakest overall results and depends most on the smoothing method used. Under Moving Average, the season  $p$ -value is 0.049. The day type  $p$ -value is 0.020, and the interaction is 0.0083. Results are a bit stronger with Moving Median. For Exponential smoothing and LOESS, all three effects become highly significant. This characteristic of the method at Node 13 matches that found in Sections 4.1.1.3 and 4.2.3. Node 13 is the most variable node from week to week, and LOESS is the least reliable smoother for it. The clustering results are the worst of all five representative nodes. Most CCRs are below 0.50. Except for Exponential smoothing under correlation-based distance, which reaches 0.58. Similarly. Silhouette scores rarely exceed 0.38. The MANOVA results support those analyzes.

#### 4.3.1.5. Overview of MANOVA Test A

Test A shows that both seasonal and day type patterns exist in the empirical load profiles. The results show differences in consistency across different nodes and smoothing methods. All five nodes have significant seasonal effects with at least three out of four smoothing methods. The agricultural nodes (Nodes 1 and 12) and the household node with the most obvious winter pattern (Node 11) show strong seasonal patterns regardless of the smoothing method used. However, Node 13 is different. Its seasonal significance varies depending on the method and is only weak under the Moving Average method.

Day type effects vary more. Node 11 does not show any clear day type effect with three out of four smoothing methods. Node 13 shows weak but noticeable day type effects when the Moving Average and Moving Median are applied. The interaction results present a similar trend. Nodes 1, 8, 12, and 13 have significant interactions with most methods. For Node 11, the interaction does not appear at the 0.05 level with Moving Average.

Across all nodes, Exponential smoothing gives much more extreme  $p$ -values than Moving Average, Moving Median, or LOESS. Exponential smoothing raises the baseline and introduces a time lag. This reason can make group differences appear larger than they are. Moving Average and Moving Median provide more consistent results. The  $p$ -values of the Moving Average and the Moving Median are more reliable baselines to interpret the effects.

Overall, the MANOVA finds a statistically significant structure in the data for all main nodes. For nodes with more detailed daily patterns, such as Nodes 1 and 12, this yields useful seasonal groupings. For household nodes with flatter profiles, such as Nodes 8 and 13, the significant MANOVA results suggest that the test is sensitive to small but steady seasonal changes.

#### 4.3.2. MANOVA Test B: Season, day type, and interaction effects on empirical residual profiles

The MANOVA results for Test B are presented in Tables 8 to 10. Each table reports the Pillai's trace  $p$ -values for the season, day type, and interaction effects. Their interaction for each node under a single smoothing method.

Table 8: MANOVA Test B for Moving Average.

| Node                  | Season      | Day Type    | Interaction |
|-----------------------|-------------|-------------|-------------|
| 1 (Agricultural, L2)  | $\approx 0$ | $\approx 0$ | $\approx 0$ |
| 8 (Household, H0)     | $\approx 0$ | $\approx 0$ | $\approx 0$ |
| 11 (Household, H0)    | $\approx 0$ | $\approx 0$ | $\approx 0$ |
| 12 (Agricultural, L1) | $\approx 0$ | $\approx 0$ | $\approx 0$ |
| 13 (Household, H0)    | $\approx 0$ | $\approx 0$ | $\approx 0$ |

*Note:*  $\approx 0$  indicates numerical underflow (below float64 precision).

##### 4.3.2.1. Hypotheses: MANOVA Test B

Test B evaluates whether empirical load profiles at individual nodes align with the BDEW SLP structure. For each daily observation, the residual profile is calculated by subtracting the corresponding BDEW SLP as follows

Table 9: MANOVA Test B for Moving Median.

| Node                  | Season                 | Day Type                | Interaction            |
|-----------------------|------------------------|-------------------------|------------------------|
| 1 (Agricultural, L2)  | $3.78 \times 10^{-53}$ | $4.55 \times 10^{-44}$  | $8.37 \times 10^{-48}$ |
| 8 (Household, H0)     | $6.12 \times 10^{-61}$ | $7.68 \times 10^{-116}$ | $1.04 \times 10^{-74}$ |
| 11 (Household, H0)    | $3.85 \times 10^{-48}$ | $3.95 \times 10^{-99}$  | $5.85 \times 10^{-64}$ |
| 12 (Agricultural, L1) | $2.35 \times 10^{-56}$ | $1.26 \times 10^{-93}$  | $1.93 \times 10^{-59}$ |
| 13 (Household, H0)    | $3.27 \times 10^{-49}$ | $1.17 \times 10^{-90}$  | $3.58 \times 10^{-55}$ |

Table 10: MANOVA Test B for LOESS.

| Node                  | Season                 | Day Type               | Interaction            |
|-----------------------|------------------------|------------------------|------------------------|
| 1 (Agricultural, L2)  | $8.32 \times 10^{-47}$ | $4.88 \times 10^{-43}$ | $4.63 \times 10^{-53}$ |
| 8 (Household, H0)     | $3.21 \times 10^{-55}$ | $2.35 \times 10^{-75}$ | $1.97 \times 10^{-77}$ |
| 11 (Household, H0)    | $7.16 \times 10^{-56}$ | $5.07 \times 10^{-86}$ | $8.40 \times 10^{-56}$ |
| 12 (Agricultural, L1) | $3.83 \times 10^{-63}$ | $3.20 \times 10^{-94}$ | $5.63 \times 10^{-64}$ |
| 13 (Household, H0)    | $1.39 \times 10^{-43}$ | $1.40 \times 10^{-69}$ | $1.19 \times 10^{-60}$ |

$$\mathbf{R}_{r,s,\delta} = \mathbf{Y}_{r,s,\delta} - \mathbf{SLP}_{s,\delta},$$

where  $\mathbf{Y}_{r,s,\delta} \in \mathbb{R}^{96}$  denotes the empirical load profile of the  $r$ -th observation in group  $(s, \delta)$  as defined in Test A,  $\mathbf{SLP}_{s,\delta} \in \mathbb{R}^{96}$  denotes the normalized BDEW SLP for season  $s$  and day type  $\delta$ , and  $\mathbf{R}_{r,s,\delta}$  denotes the resulting residual profile vector.

Let  $\boldsymbol{\nu}_{s,\delta} = \mathbb{E}[\mathbf{R}_{r,s,\delta}]$  denote the population mean residual vector for group  $(s, \delta)$ . The marginal mean vectors of the residuals are defined

$$\bar{\boldsymbol{\nu}}_s = \frac{1}{3} \sum_{\delta=1}^3 \boldsymbol{\nu}_{s,\delta}, \quad \bar{\boldsymbol{\nu}}_\delta = \frac{1}{3} \sum_{s=1}^3 \boldsymbol{\nu}_{s,\delta}, \quad \bar{\boldsymbol{\nu}} = \frac{1}{9} \sum_{s=1}^3 \sum_{\delta=1}^3 \boldsymbol{\nu}_{s,\delta}, \quad (40)$$

where  $\bar{\boldsymbol{\nu}}_s$  denotes the marginal mean residual vector for season  $s$ ,  $\bar{\boldsymbol{\nu}}_\delta$  denotes the marginal mean residual vector for day type  $\delta$ , and  $\bar{\boldsymbol{\nu}}$  is the overall grand mean residual vector. Then, the following hypotheses are tested using Pillai's trace.

**Season effect:**

$$H_0^{(S)} : \bar{\boldsymbol{\nu}}_{s_1} = \bar{\boldsymbol{\nu}}_{s_2} = \bar{\boldsymbol{\nu}}_{s_3}$$

$$H_1^{(S)} : \text{not all } \bar{\boldsymbol{\nu}}_{s_1} = \bar{\boldsymbol{\nu}}_{s_2} = \bar{\boldsymbol{\nu}}_{s_3}$$

Table 11: MANOVA Test B for Exponential Smoothing.

| Node                  | Season                  | Day Type                | Interaction             |
|-----------------------|-------------------------|-------------------------|-------------------------|
| 1 (Agricultural, L2)  | $4.53 \times 10^{-166}$ | $1.07 \times 10^{-144}$ | $3.93 \times 10^{-246}$ |
| 8 (Household, H0)     | $1.12 \times 10^{-147}$ | $1.50 \times 10^{-251}$ | $3.27 \times 10^{-256}$ |
| 11 (Household, H0)    | $2.41 \times 10^{-140}$ | $4.80 \times 10^{-275}$ | $9.61 \times 10^{-267}$ |
| 12 (Agricultural, L1) | $1.77 \times 10^{-188}$ | $1.07 \times 10^{-232}$ | $4.97 \times 10^{-257}$ |
| 13 (Household, H0)    | $2.18 \times 10^{-166}$ | $3.96 \times 10^{-221}$ | $1.73 \times 10^{-233}$ |

**Day type effect:**

$$H_0^{(\delta)} : \bar{\nu}_{\delta_1} = \bar{\nu}_{\delta_2} = \bar{\nu}_{\delta_3}$$

$$H_1^{(\delta)} : \text{not all } \bar{\nu}_{\delta_1} = \bar{\nu}_{\delta_2} = \bar{\nu}_{\delta_3}$$

**Season  $\times$  day type interaction:**

$$H_0^{(S \times \delta)} : \nu_{s,\delta} - \bar{\nu}_s - \bar{\nu}_\delta + \bar{\nu} = \mathbf{0} \quad \forall s, \delta$$

$$H_1^{(S \times \delta)} : \exists (s, \delta) \text{ such that } \nu_{s,\delta} - \bar{\nu}_s - \bar{\nu}_\delta + \bar{\nu} \neq \mathbf{0}$$

where each hypothesis is evaluated independently and the corresponding  $p$ -values are reported for each node and smoothing method in Tables method in Tables 8 to 10.

#### 4.3.2.2. MANOVA Test B of agriculture representative node: Node 1

Node 1 demonstrates highly significant results for all three effects across all smoothing methods. The Moving Average shows  $p$ -values at numerical underflow for season, day type, and interaction effects. The Moving Median shows  $p$ -values of  $3.78 \times 10^{-33}$ ,  $4.55 \times 10^{-44}$ , and  $8.37 \times 10^{-53}$  for season, day type, and interaction effects, respectively. LOESS also presents strong results, with the  $p$ -value of season effect at  $8.32 \times 10^{-67}$ . Exponential smoothing produces the most extreme values, with all three effects below  $10^{-200}$ .

These outcomes are consistent with the mean empirical load profile analysis presented in Section 4.1.2.1. The SLP of L2 consistently overestimates the empirical mean at Node 1 across all seasons, day types, and smoothing methods. The SLP remains above the uncertainty boundary of the observed data in the summer and transition on weekdays. This shows that all observed daily variations of Node1 is below the SLP prediction. Clustering results in Section 4.2.1 further support this conclusion. CCRs above 0.50 are observed for most distance metrics and smoothing methods. Thus, the presence of seasonal structure is identified at this node. The seasonal pattern persists in the residuals after SLP subtraction. Consequently, the L2 standard load profile does not accurately represent the actual consumption at this agricultural node.

#### 4.3.2.3. MANOVA Test B of household representative node: Node 8

Node 8 demonstrates strong statistical significance in all cases. The season effect  $p$ -values are between  $3.21 \times 10^{-50}$  and  $1.12 \times 10^{-256}$  for Moving Median and Exponential smoothing, respectively. Day type and interaction effects are also highly significant in every test. The underlying cause of this strong result is evident from the mean empirical load profile analysis in Section 4.1.2.2. The empirical mean of this node exhibits neither a morning nor an evening peak. In contrast, the H0 SLP exhibits a characteristic double peak shape. After subtracting this peaked SLP from the nearly flat empirical profiles, the inverted peak of the SLP shows up in the residuals as a large negative feature. This feature retains the full seasonal and day type structure of the SLP. Hence, the MANOVA is identified as highly significant.

The observed significance does not indicate the presence of seasonal structure in the empirical data. Instead, it reflects the structure of the SLP that is absent from the empirical data. The clustering analysis in Section 4.2.2 supports this conclusion from a different perspective. Silhouette scores remain consistently below 0.50 across all distance metrics. This indicates that the empirical profiles at this node lack coherent within-season clustering signals. Both Euclidean and correlation-based distance metrics show CCR values above 0.50. Therefore, the residual MANOVA suggests that the H0 standard load profile does not align with the actual active power time series behavior for Node 8.

#### 4.3.2.4. MANOVA Test B of remaining nodes

Node 11 (Household, H0) shows the most distinctive pattern in the residual MANOVA. The season effect is highly significant for all smoothing methods. The  $p$ -values are from  $3.85 \times 10^{-48}$  under Moving Median to  $2.41 \times 10^{-140}$  under Exponential smoothing.. However, the day type effect of Moving Average, Moving Median, or LOESS is not significant. All four methods give  $p$ -values of approximately zero for day type. This indicates highly significant effects across all methods. Similarly, all interaction effects underflow to approximately zero for all smoothing methods. This pattern suggests the residuals do not differ systematically between weekdays, Saturdays, and Sundays. Consequently, the deviation of Node 11 from the H0 SLP is largely driven by seasonal effects rather than by the type of day. This is corroborated by the mean load profile analysis in Section 4.1.2.3. Node 11 displays slightly more intraday structure than Nodes 8 and 13. Nevertheless, the SLP remains well above the empirical mean across all day types and seasons. Thus, this aligns with the nonsignificant day type result.

The Exponential smoothing results for this node show a notable day type pattern. Section 4.1.1.3 shows that this method produces an elevated baseline and amplified seasonal contrasts. These properties likely inflate the day type signal beyond what the underlying data support. Hence, the day type effects of the Moving Average and Moving Median

are not significant. The clustering results in Section 4.2.3 add a further perspective. The CCR values attain their highest levels under DTW for both the Moving Average and the Moving Median methods. However, the two findings address different dimensions. The clustering captures seasonal differences in profile level and shape, while the MANOVA tests whether the deviation from the H0 SLP varies by day type after accounting for the seasonal component. At Node 11, no such differences are observed.

Node 12 illustrates highly significant results. All  $p$ -values are extremely small in every case. The mean load profile analysis in Section 4.1.2.3 showed that the SLP of L1 fits the morning and evening peak structure of Node 12 more closely than the SLP of L2. Furthermore, the uncertainty bands are wide, and the SLP does not consistently fall within the observed range. The residual MANOVA confirms that the departure from the L1 standard is structured along both seasonal and day type dimensions.

Node 13 shows the weakest residual MANOVA results for Moving Average and Moving Median. The season effect of Moving Average underflows to approximately zero. Under Moving Median, the season  $p$ -value is  $3.27 \times 10^{-49}$ . Day type and interaction effects remain significant for both methods. Exponential smoothing and LOESS produce highly significant results for all three effects at this node. This contrast across smoothing methods is fully consistent with the pattern established in Sections 4.1 and 4.2. Among the five nodes, Node 13 demonstrates the highest between week variability. Its uncertainty bands are the widest relative to other methods. In addition, it also shows poor clustering performance. The CCR values fall below 0.50 in most configurations. The DTW Silhouette score is as low as 0.09 for LOESS on Sundays. The empirical profiles at this node are highly irregular. After SLP subtraction, the residuals do not form a cleanly structured pattern. In particular, the Moving Average and Moving Median exhibit only a weak seasonal effect. Exponential smoothing and LOESS show stronger and more extreme  $p$ -values than other household nodes for Moving Average or Moving Median. This aligns with previous analyses, which show that LOESS tends to overfit local clusters of anomalous weeks. On the other hand, Exponential smoothing tends to make seasonal differences appear larger than they really are.

#### **4.3.2.5. Overview of MANOVA Test B**

The residual MANOVA results confirm the earlier statistical analysis. The residuals show significant effects for season, day type, and their interaction in every test. All nodes and all smoothing methods yield  $p$ -values that underflow to approximately zero for every effect. This is consistent with the finding that the empirical profiles at these nodes differ systematically from their respective SLPs across all seasons and day types.

The  $p$ -values from Exponential smoothing are much more extreme than those from Moving Average and Moving Median for all nodes. This only confirms significance without

providing additional insights. The pattern aligns with the known characteristics of Exponential smoothing described in Section 4.1.1.3. For interpreting the conformance test, the results from the Moving Average and the Moving Median are more trustworthy. Using these two methods, the main conclusion holds for every node.

In summary, Agricultural nodes reveal patterns that do not fit the L1 or L2 structure. On the other hand, household nodes exhibit highly variable profiles that differ significantly from the typical double-peak shape of the H0 profile. Consequently, the actual active power time series at these SimBench nodes do not follow the BDEW SLP structure at the individual node level.

## 5. Conclusion

This thesis investigated whether the active power time series at five representative nodes of the SimBench low-voltage distribution grid in a rural area are consistent with the BDEW SLPs. The five nodes included two agricultural nodes (Nodes 1 and 12, assigned to L2 and L1 profiles, respectively) and three household nodes (Nodes 8, 11, and 13, all assigned to H0). The analysis proceeded in three stages. These are sequential two-stage smoothing to derive empirical load profiles, K-means clustering to assess seasonal separability, and two MANOVA tests to test the effects of season and day type before and after subtracting the corresponding SLPs.

The visual comparison of mean empirical load profiles against the BDEW SLPs revealed a persistent and substantial gap at all five nodes. The SLP consistently overestimated per-unit active power consumption. In most panels, the SLP exceeded the upper boundary of the empirical uncertainty band. This pattern is explained by the difference in scale. The standard load profile is constructed by averaging normalized consumption across many real consumers. Thus, it retains high variability. Among the five nodes, Node 12 showed the closest visual alignment with the L1 SLP. This was particularly visible in the bimodal morning and evening peak structure. Household nodes displayed nearly flat empirical means with no recognizable double-peak shape.

The clustering results reflected the same structure. Silhouette scores were generally low across all nodes, distance metrics, and smoothing methods. The results suggested that the seasonal clusters are not clearly separated in the empirical load profiles space. Agricultural nodes, especially Node 1, achieved CCR values above 0.50 in most configurations. The highest value reached 0.90. This result was seen with both Euclidean distance and correlation-based distance when using Exponential smoothing. Node 13 showed poor clustering results, with CCR values mostly below 0.5. Similarly, Node 8 presented the low Silhouette scores among all five nodes across all configurations. The Euclidean distance



proved to be most effective when the seasonal signal was primarily a shift in consumption levels. The correlation-based distance performed comparably to Euclidean distance. The models demonstrated strong performance for nodes exhibiting distinct intraday pattern variations across different seasons. Overall, the clustering results suggest that the empirical load profiles do not consistently recover the season and day type grouping defined by the BDEW SLPS.

MANOVA Test A confirmed the presence of significant season and day type effects in the empirical load profiles at all five nodes. The agricultural nodes and Node 11 showed strong statistical significance across all four smoothing methods. In contrast, Node 13 showed only weak seasonal significance. This was observed only under Moving Average and Moving Median. MANOVA Test B indicated that these effects persisted. Season, day type, and interaction effects were highly significant for all nodes and smoothing methods. If the empirical load profiles followed the structure of the SLPs, subtracting these profiles would remove seasonal and day type patterns from the residuals. Unfortunately, these patterns remain significant.

In summary, Node 12 most closely shared the L1 SLP in profile shape among the five nodes. Its bimodal morning and evening peak structure was visually recognizable. However, this alignment is limited to the smoothing results. Both clustering and MANOVA suggested that Node 12 still differs significantly from the SLP. In contrast, the other agricultural node (Node 1) and household nodes did not show any recognizable SLP shape.

## 6. Research limitations

Several limitations for future research should be acknowledged. This analysis focused exclusively on active power (P). Reactive power (Q) could be examined using the same framework. Another point is that only nine nationwide public holidays were excluded. Regional holidays were not considered. Future research could investigate whether aggregating a larger set of nodes in the SimBench network yields profiles more closely aligned with the SLPs. Aggregation across multiple consumers is fundamental to SLP construction. Furthermore, the assumption of equal covariance matrices in MANOVA was not met. The independence of daily observations cannot be fully ensured because of the temporal structure of the data. These limitations should be considered when interpreting the test results.

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## A. Appendix

### A.1. Additional figures

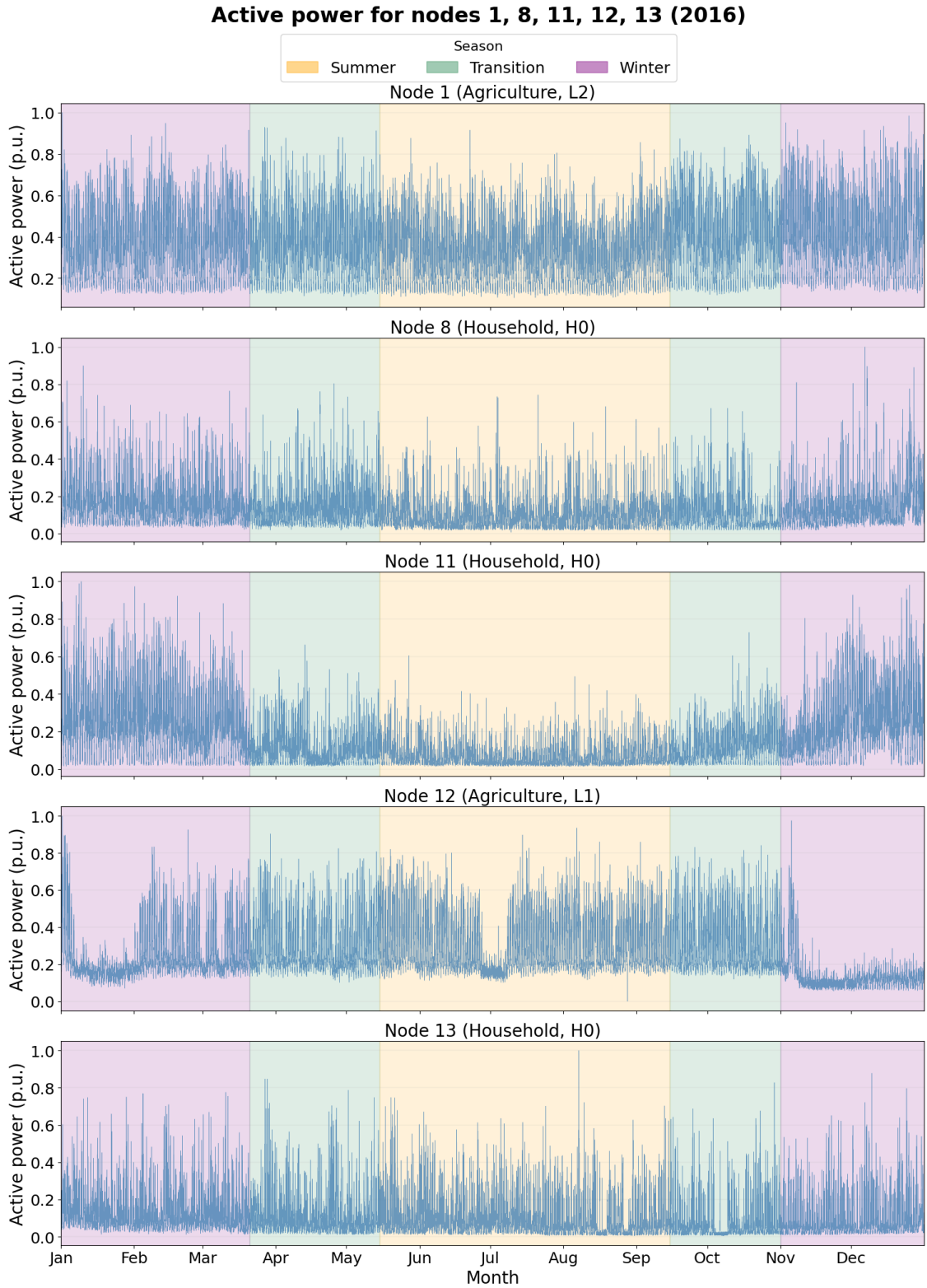
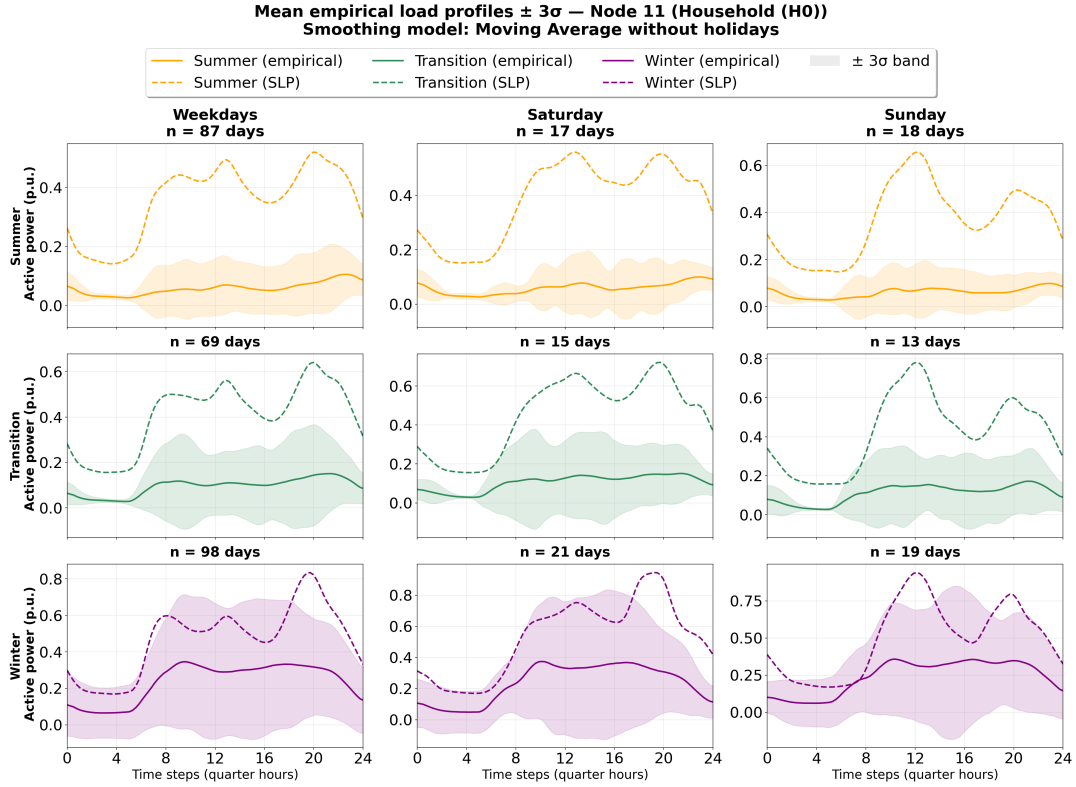
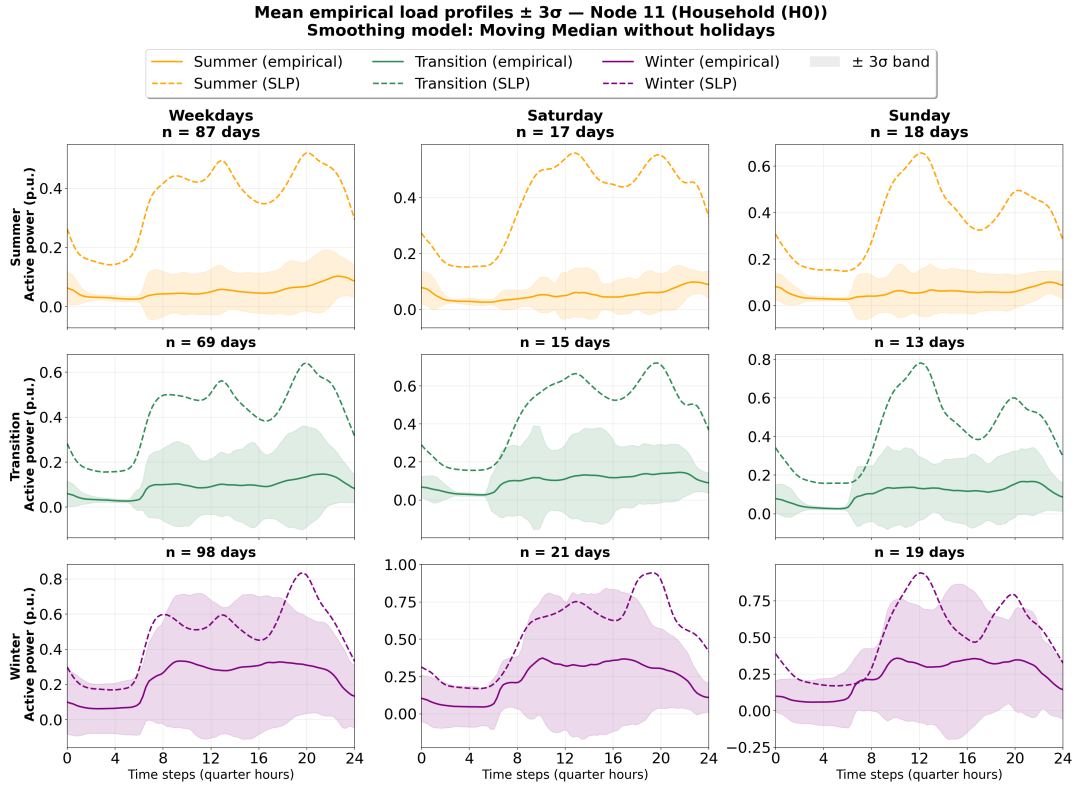


Figure 26: Quarter-hourly active power time series for nodes 1, 8, 11, 12, and 13 across 2016

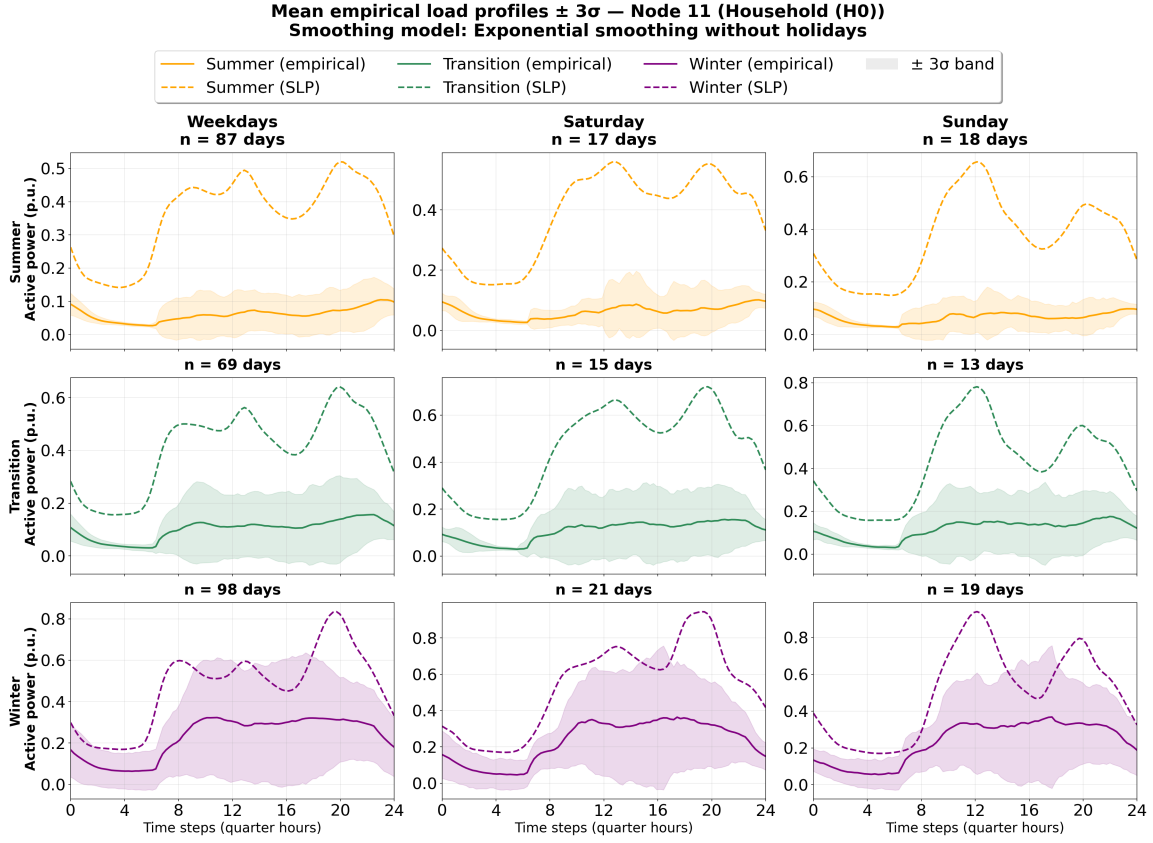
## A.2. Additional figures of mean empirical profiles and SLPs



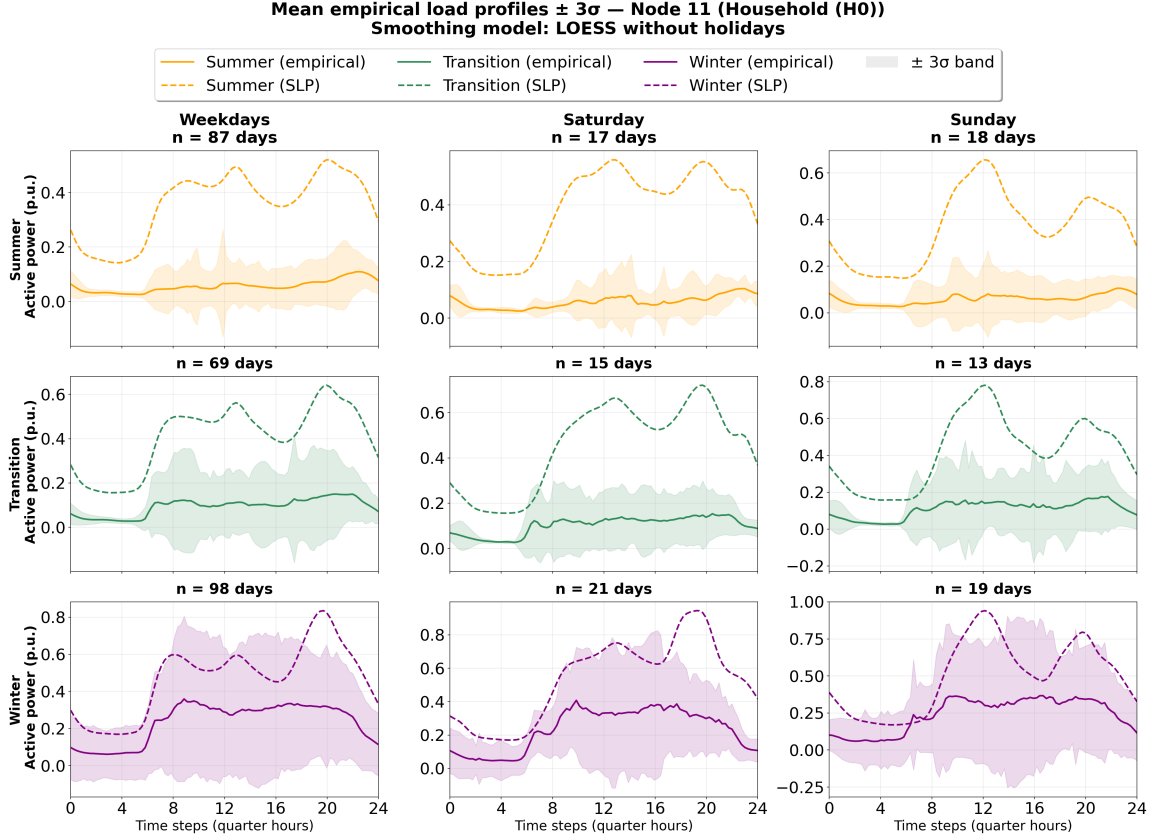
(a) Mean empirical load profiles  $\pm 3\sigma$  for Node 11 (H0) smoothed with Moving Average (without holidays).



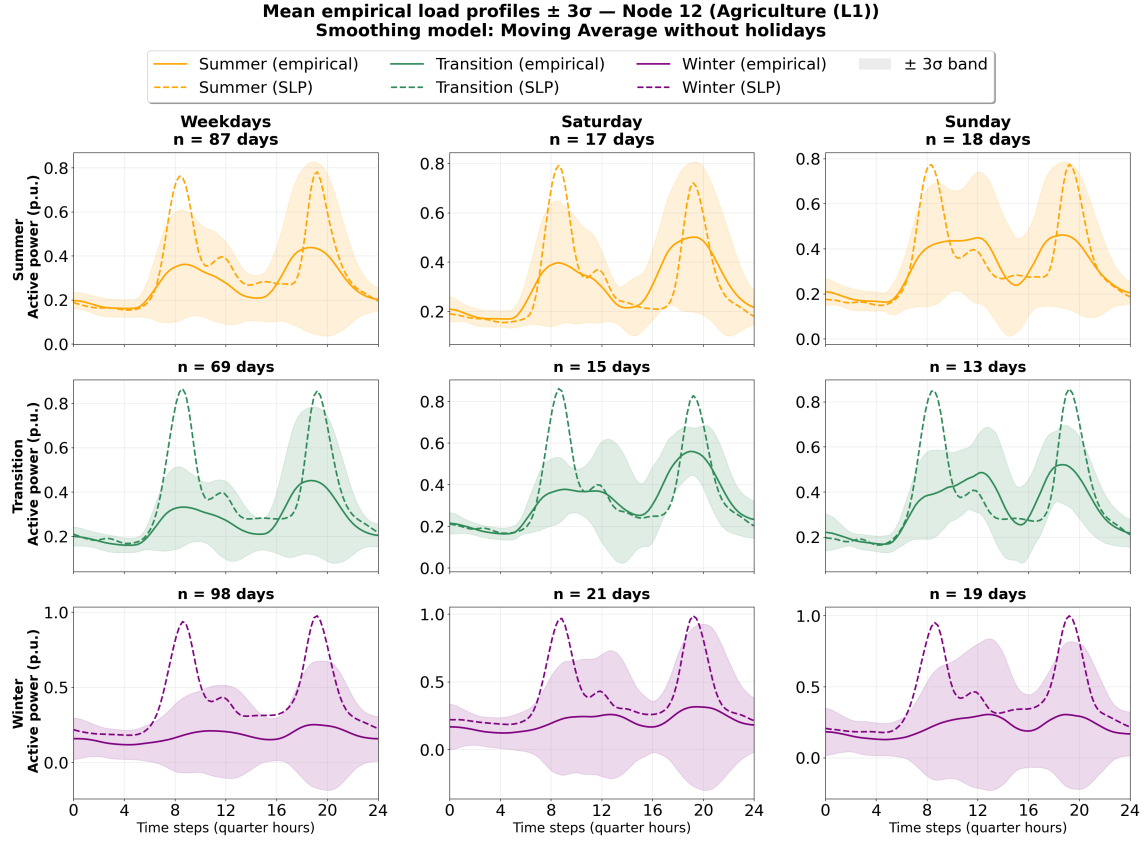
(b) Mean empirical load profiles  $\pm 3\sigma$  for Node 11 (H0) smoothed with Moving Median (without holidays).



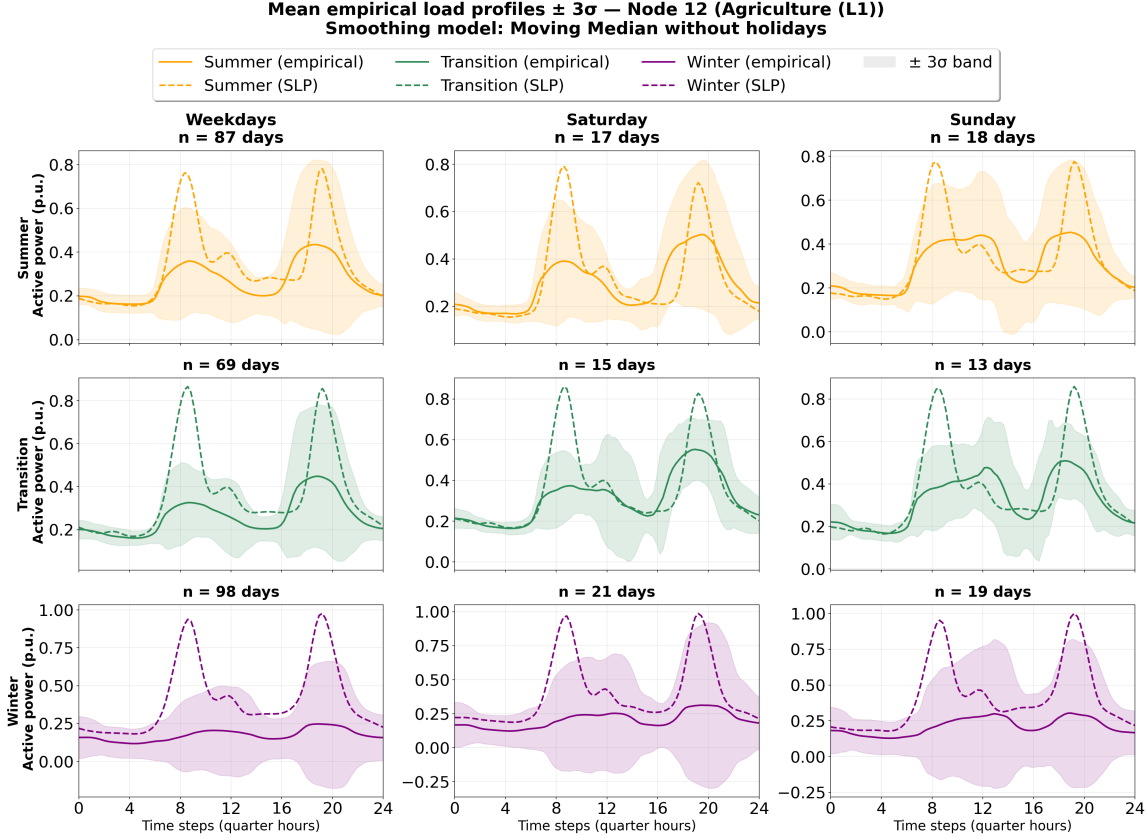
(a) Mean empirical load profiles  $\pm 3\sigma$  for Node 11 (Household, H0) smoothed with Exponential smoothing (without holidays).



(b) Mean empirical load profiles  $\pm 3\sigma$  for Node 11 (Household, H0) smoothed with LOESS (without holidays).

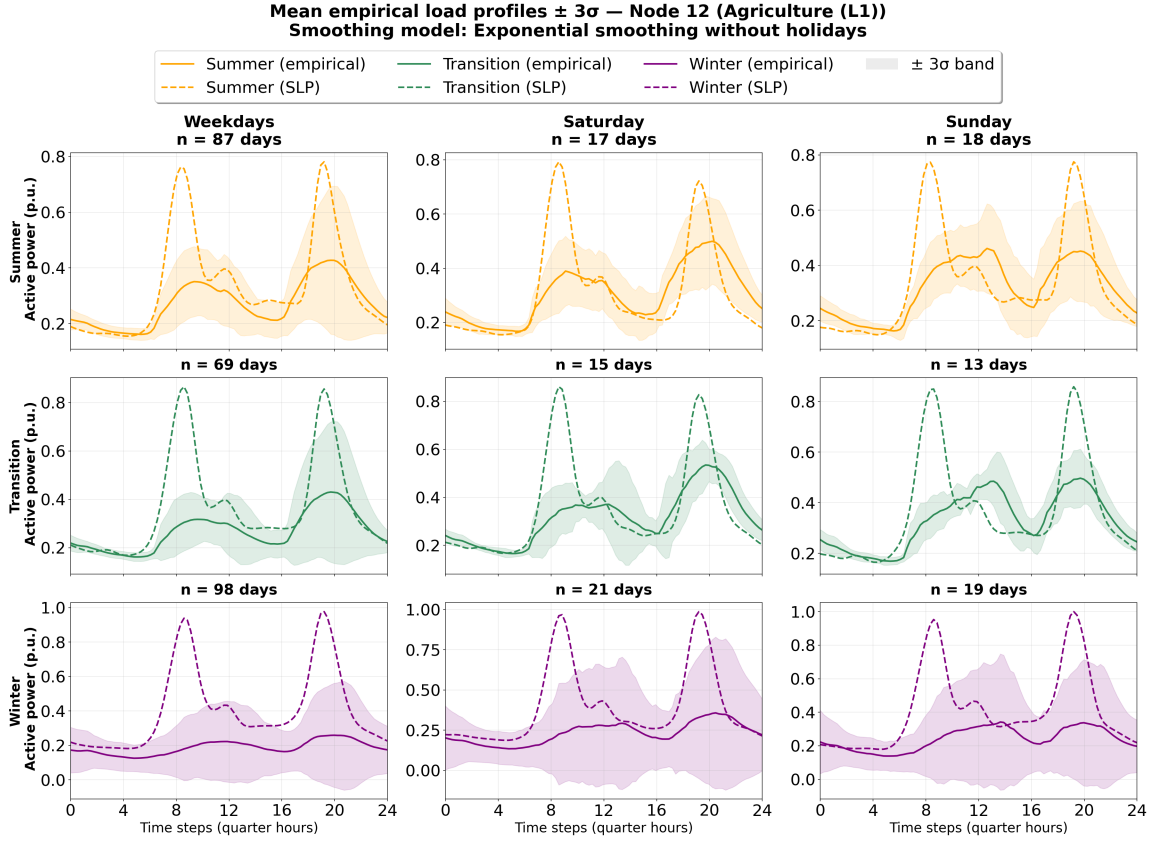


(a) Mean empirical load profiles  $\pm 3\sigma$  for Node 12 (Agriculture, L1) smoothed with Moving Average (without holidays).

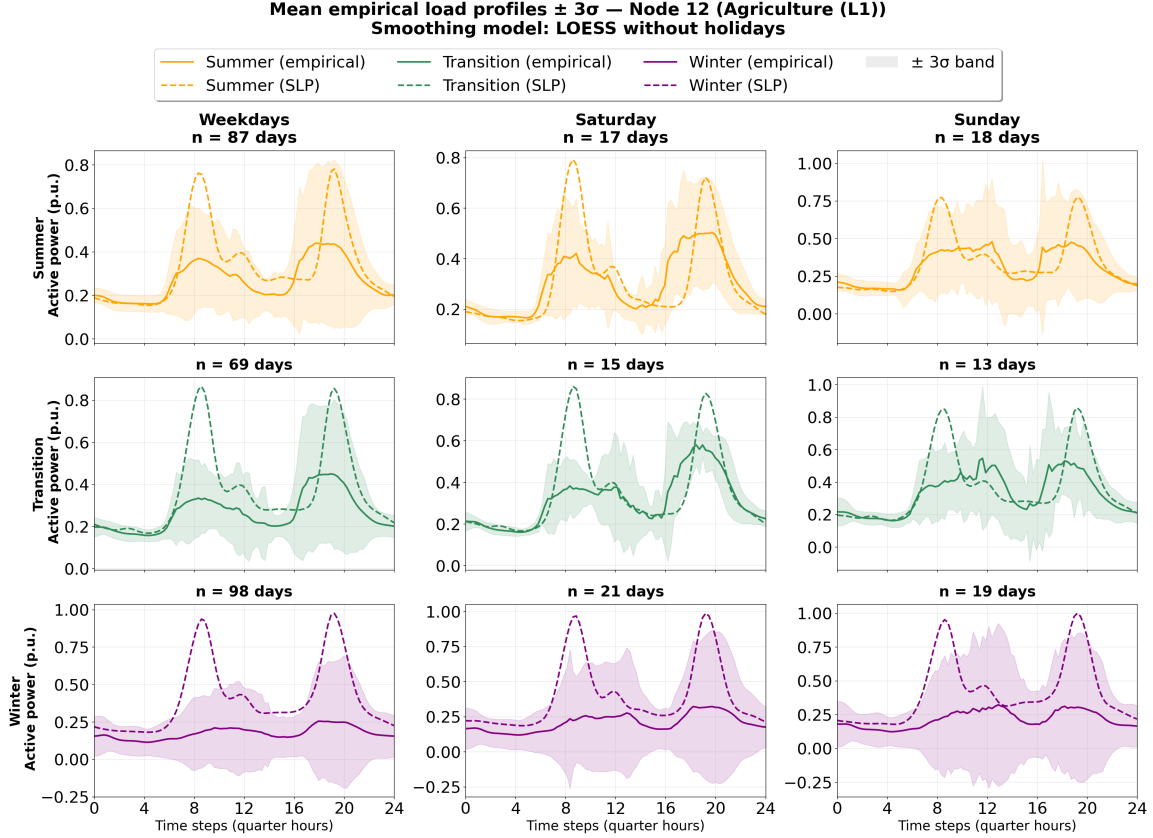


(b) Mean empirical load profiles  $\pm 3\sigma$  for Node 12 (Agriculture, L1) smoothed with Moving Median (without holidays).

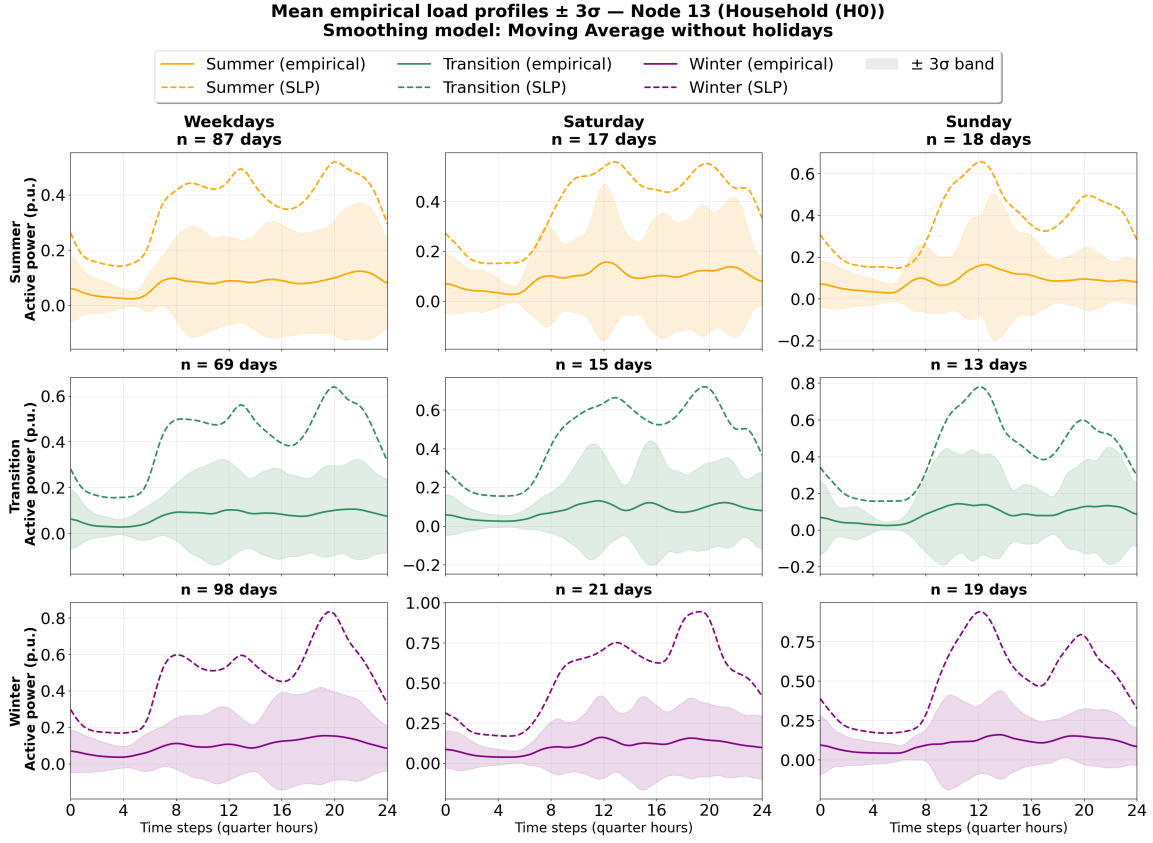




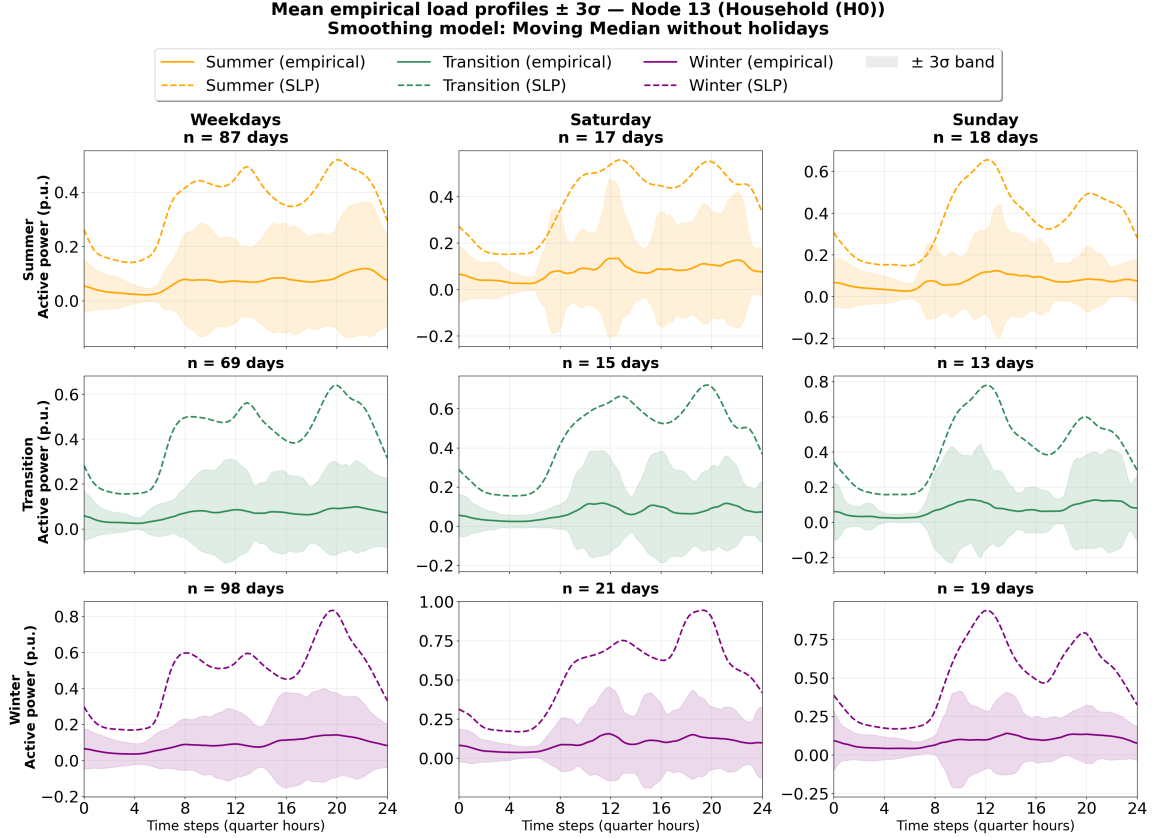
(a) Mean empirical load profiles  $\pm 3\sigma$  for Node 12 (Agriculture, L1) smoothed with Exponential smoothing (without holidays).



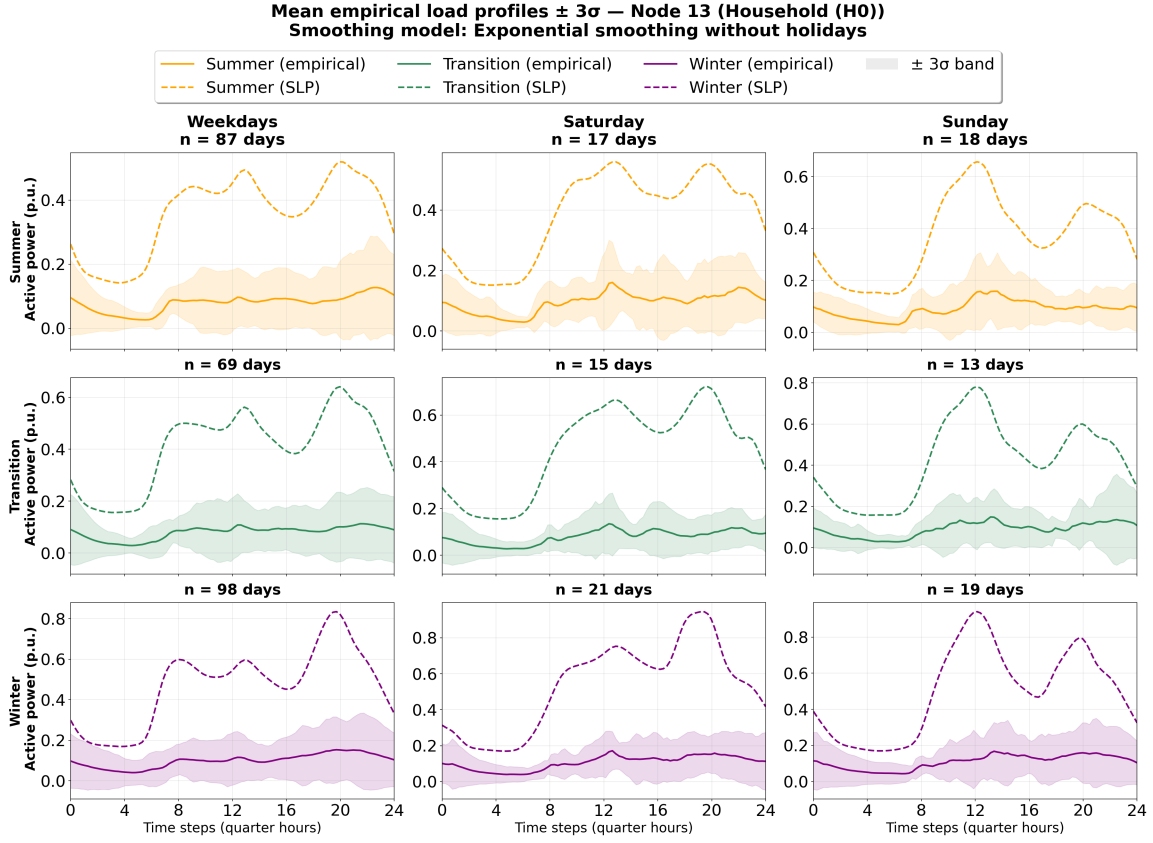
(b) Mean empirical load profiles  $\pm 3\sigma$  for Node 12 (Agriculture, L1) smoothed with LOESS (without holidays).



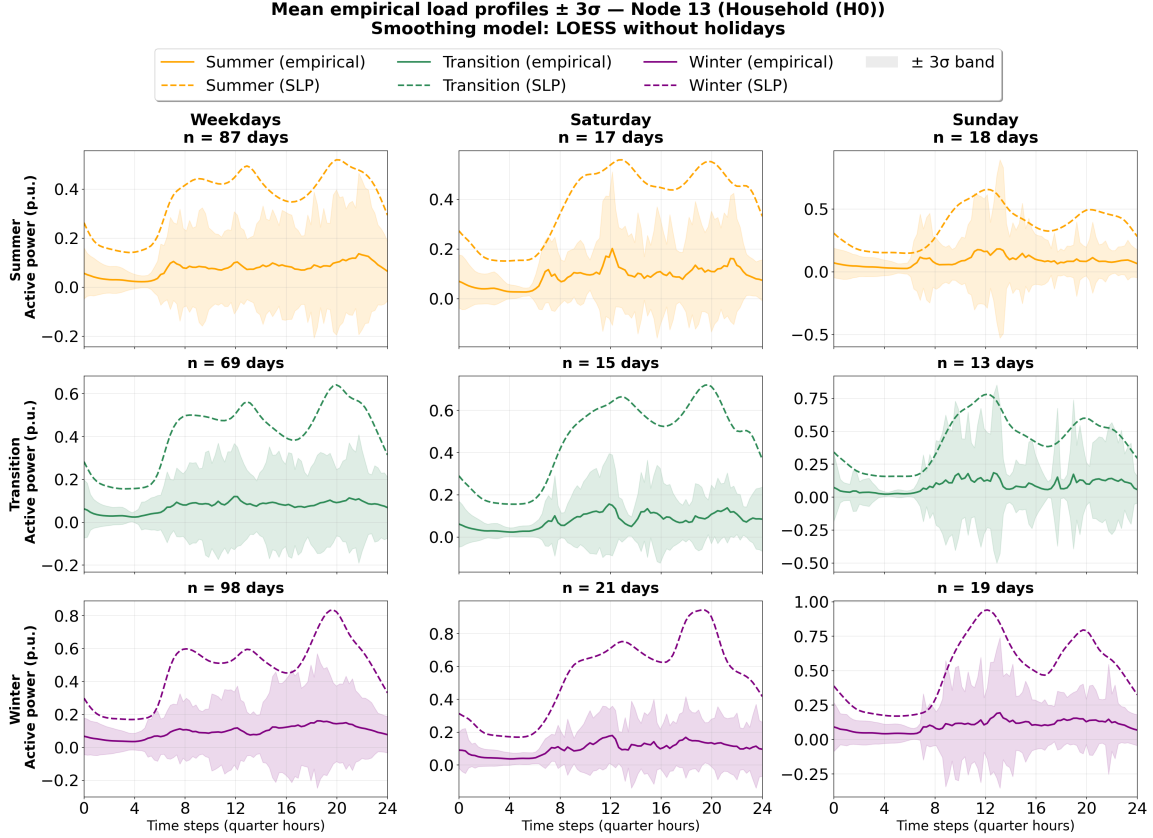
(a) Mean empirical load profiles  $\pm 3\sigma$  for Node 13 (Household, H0) smoothed with Moving Average (without holidays).



(b) Mean empirical load profiles  $\pm 3\sigma$  for Node 13 (Household, H0) smoothed with Moving Median (without holidays).



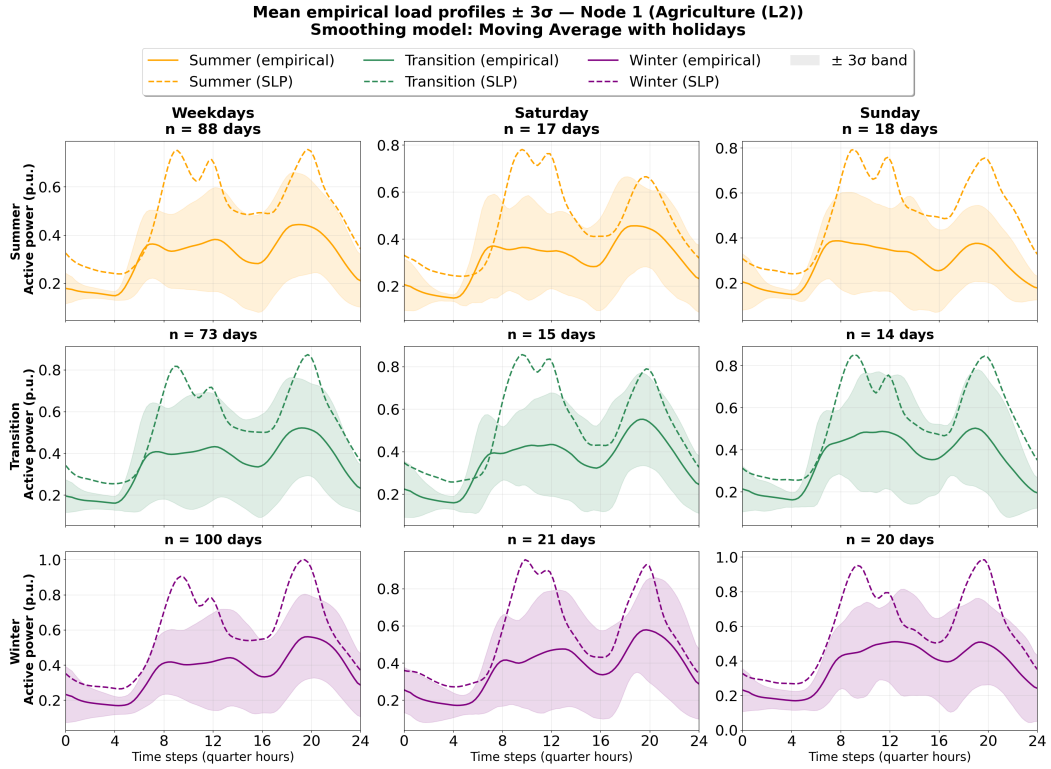
(a) Mean empirical load profiles  $\pm 3\sigma$  for Node 13 (Household, H0) smoothed with Exponential smoothing (without holidays).



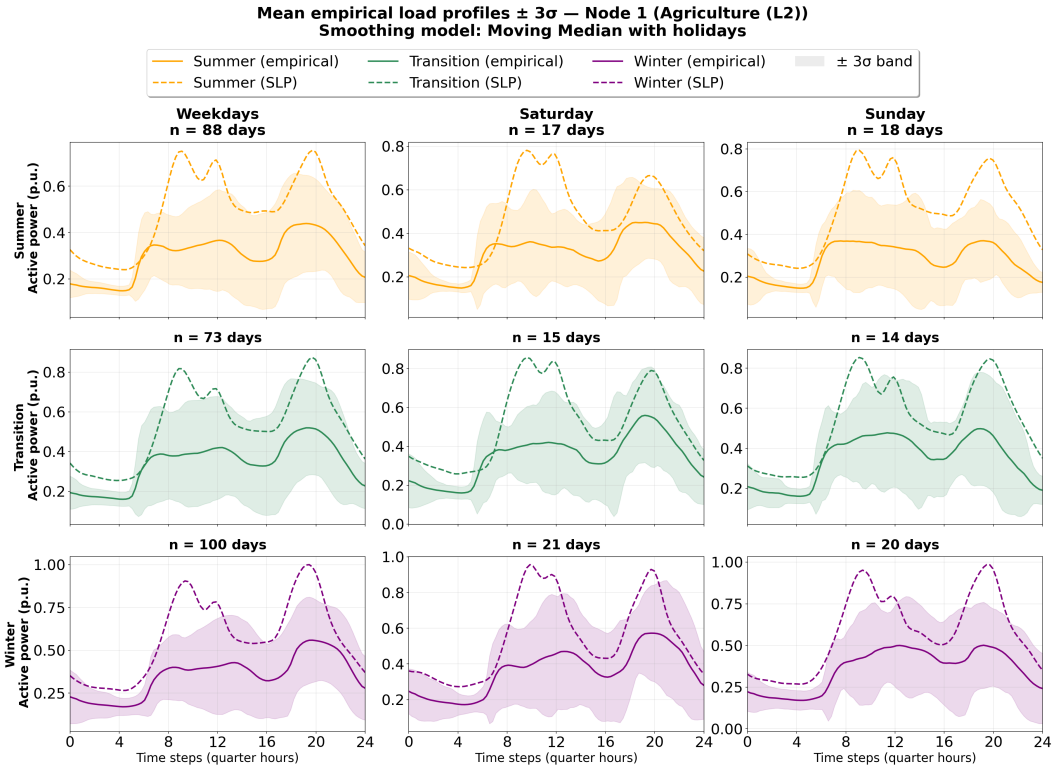
(b) Mean empirical load profiles  $\pm 3\sigma$  for Node 13 (Household, H0) smoothed with LOESS (without holidays).



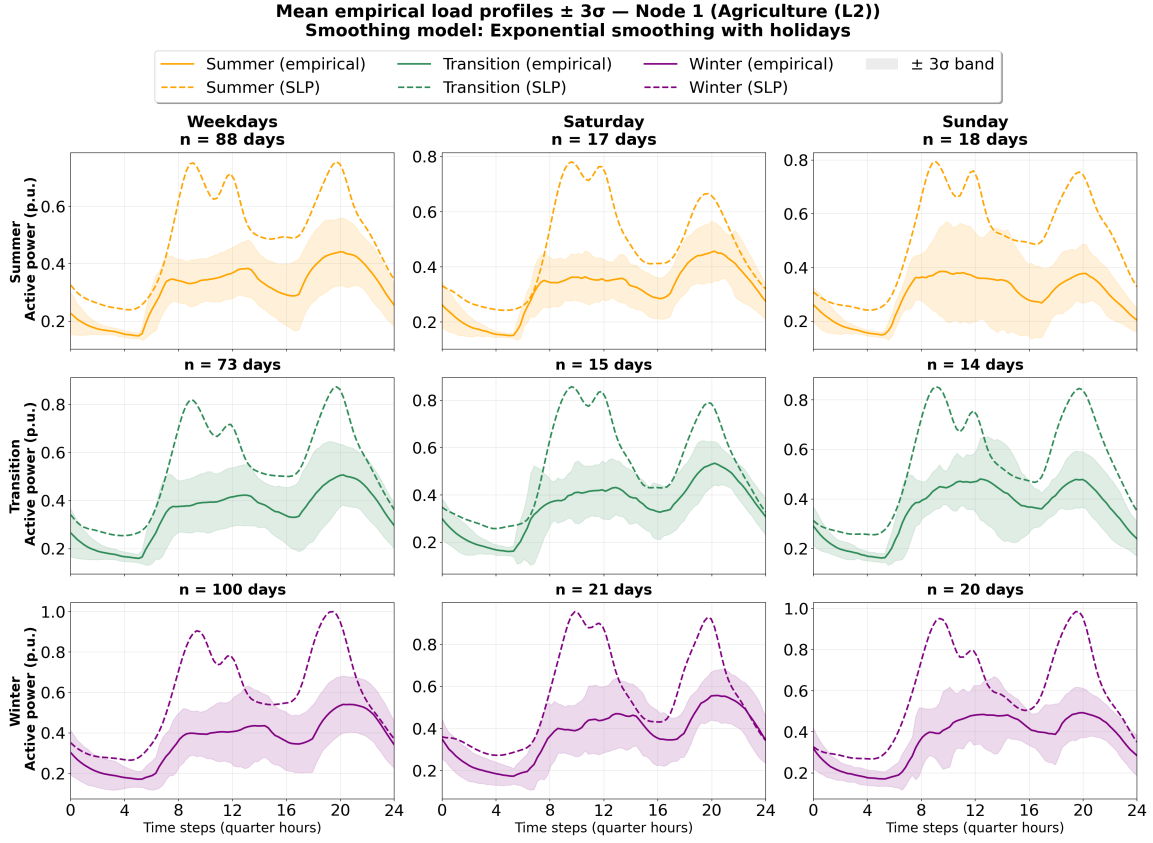
### A.3. Additional figures of stage 2 smoothed load profiles with holidays and SLPs



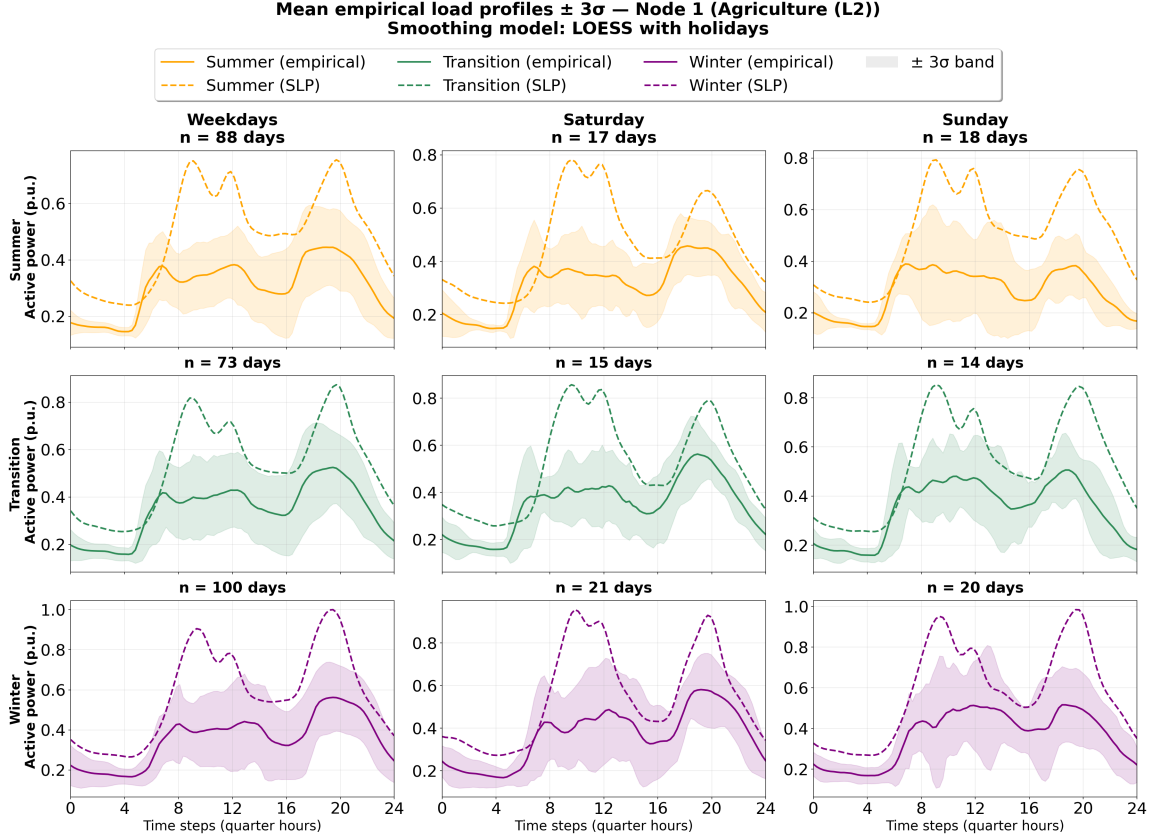
(a) Mean empirical load profiles  $\pm 3\sigma$  for Node 1 (Agriculture, L2) smoothed with Moving Average (with holidays).



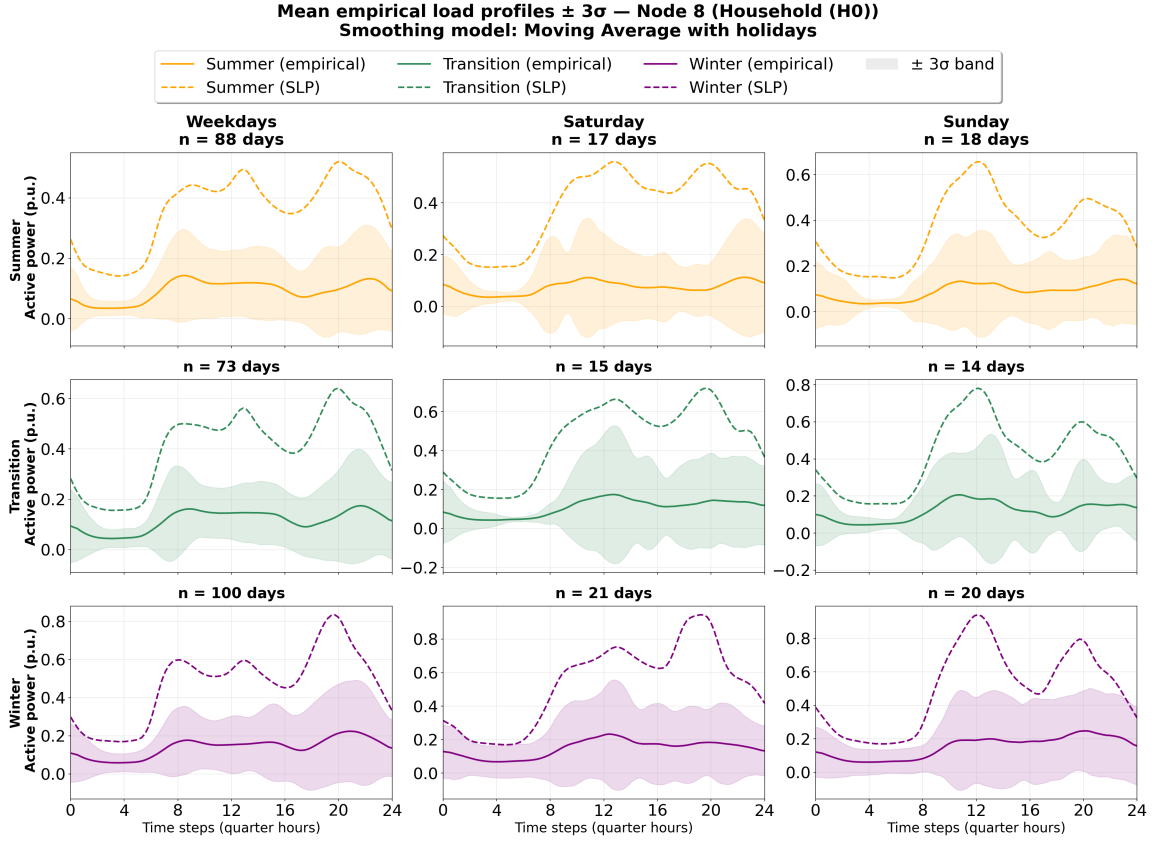
(b) Mean empirical load profiles  $\pm 3\sigma$  for Node 1 (Agriculture, L2) smoothed with Moving Median (with holidays).



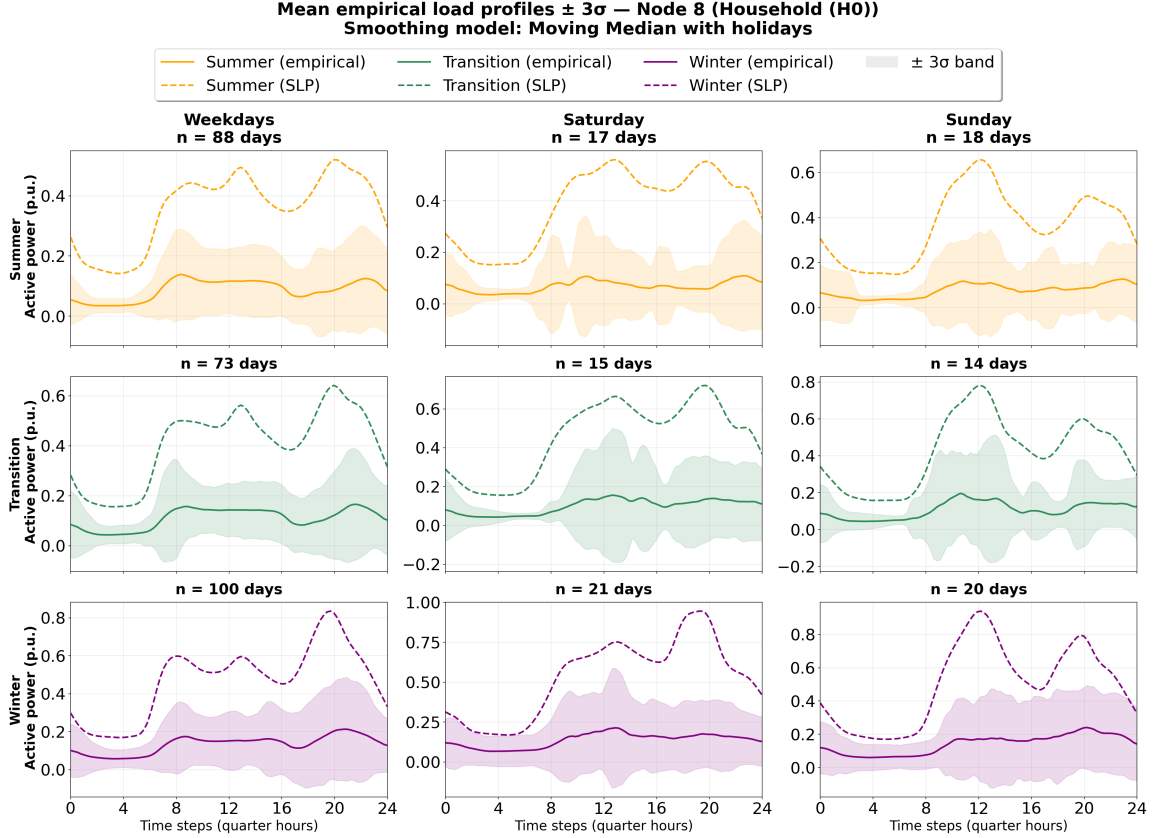
(a) Mean empirical load profiles  $\pm 3\sigma$  for Node 1 (Agriculture, L2) smoothed with Exponential smoothing (with holidays).



(b) Mean empirical load profiles  $\pm 3\sigma$  for Node 1 (Agriculture, L2) smoothed with LOESS (with holidays).

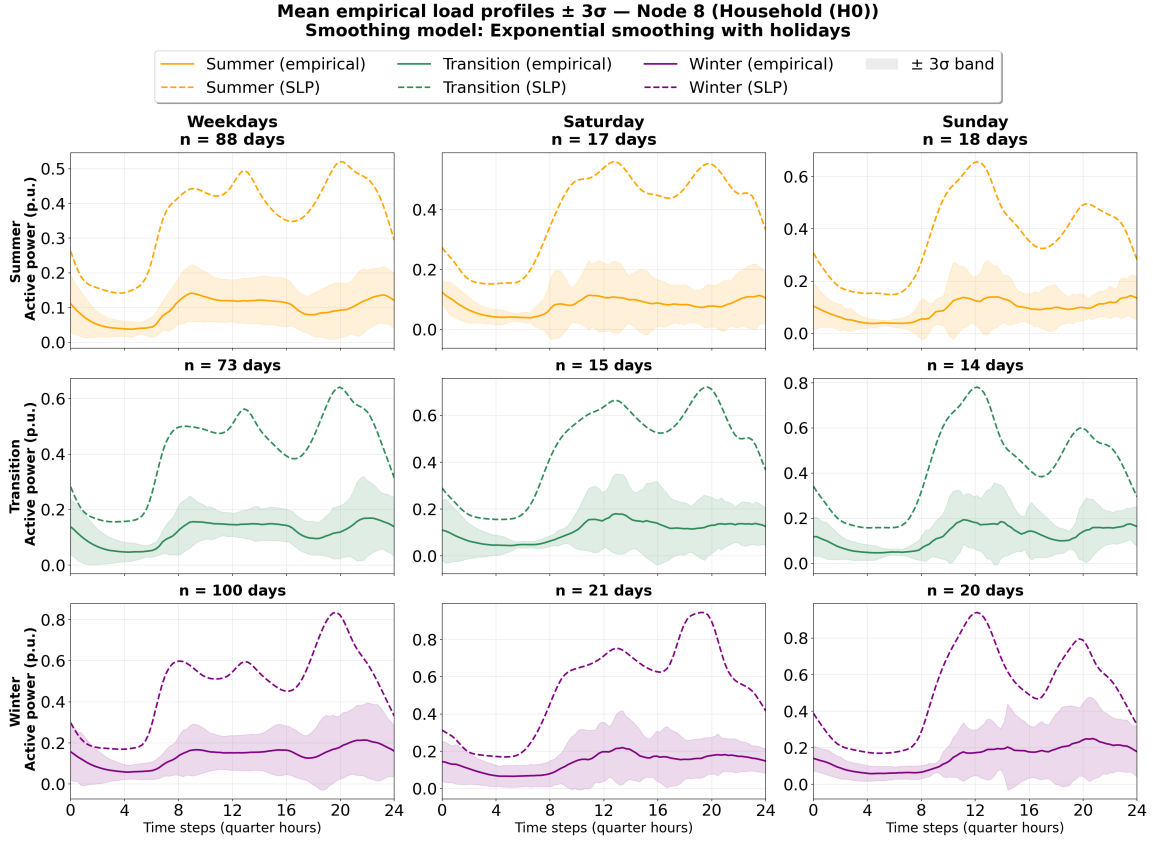


(a) Mean empirical load profiles  $\pm 3\sigma$  for Node 8 (Household, H0) smoothed with Moving Average (with holidays).

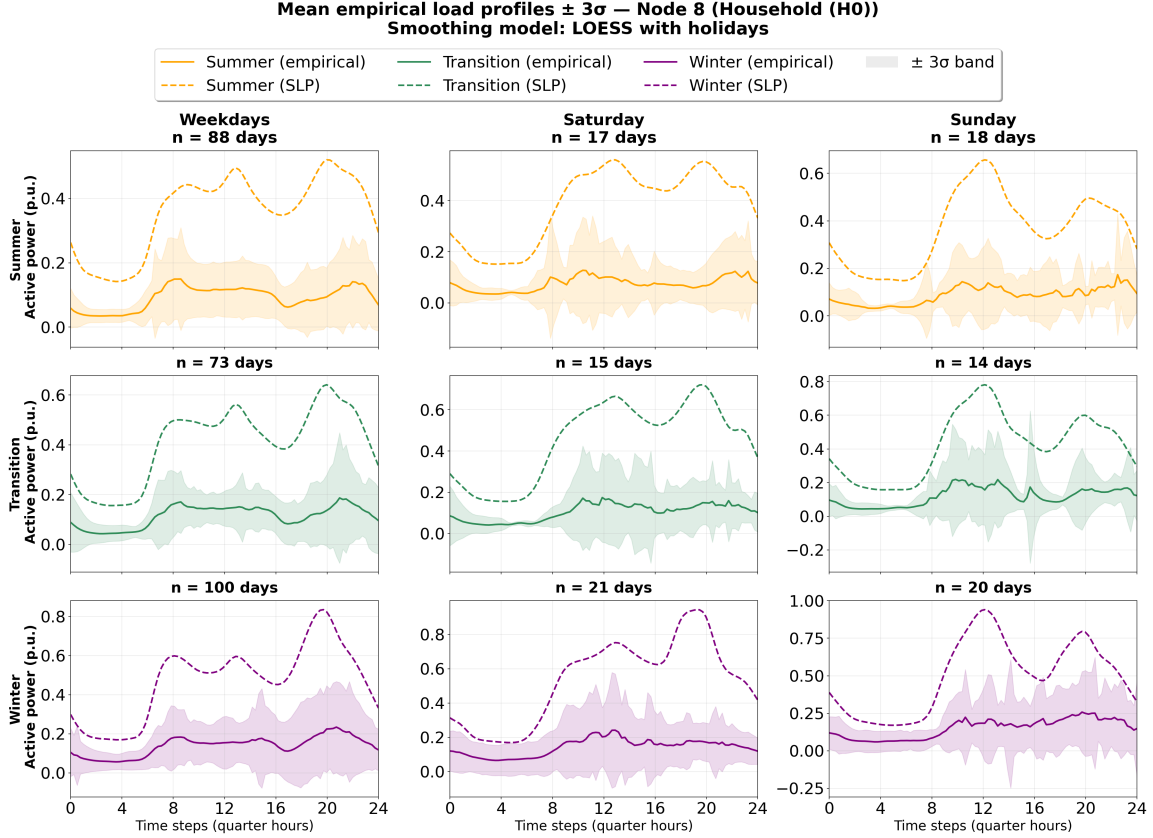


(b) Mean empirical load profiles  $\pm 3\sigma$  for Node 8 (Household, H0) smoothed with Moving Median (with holidays).





(a) Mean empirical load profiles  $\pm 3\sigma$  for Node 8 (Household, H0) smoothed with Exponential smoothing (with holidays).

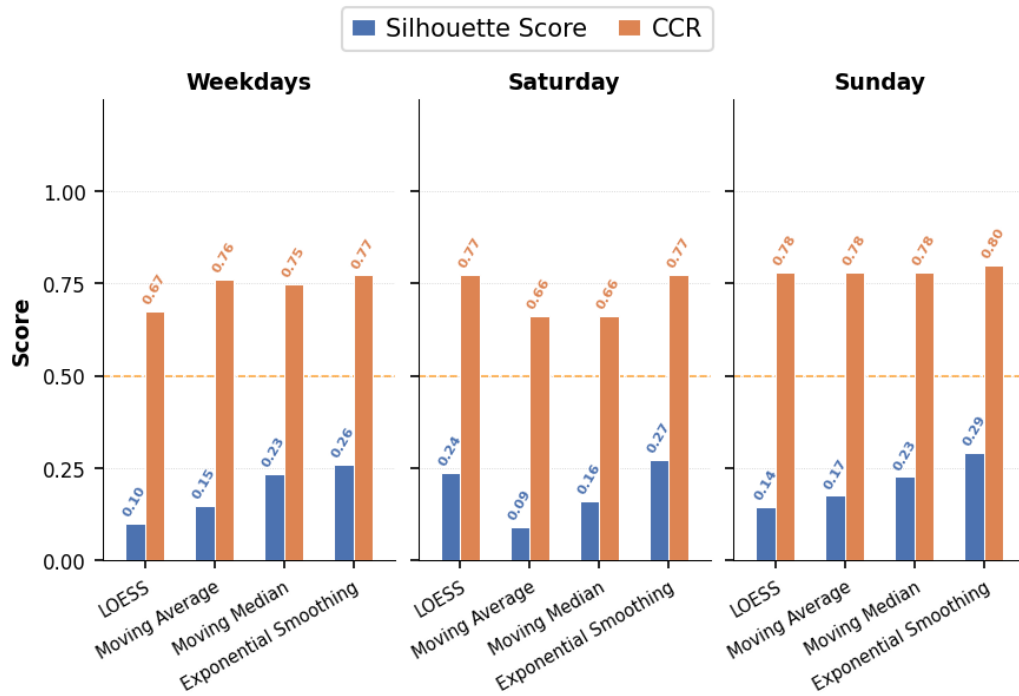


(b) Mean empirical load profiles  $\pm 3\sigma$  for Node 8 (Household, H0) smoothed with LOESS (with holidays).



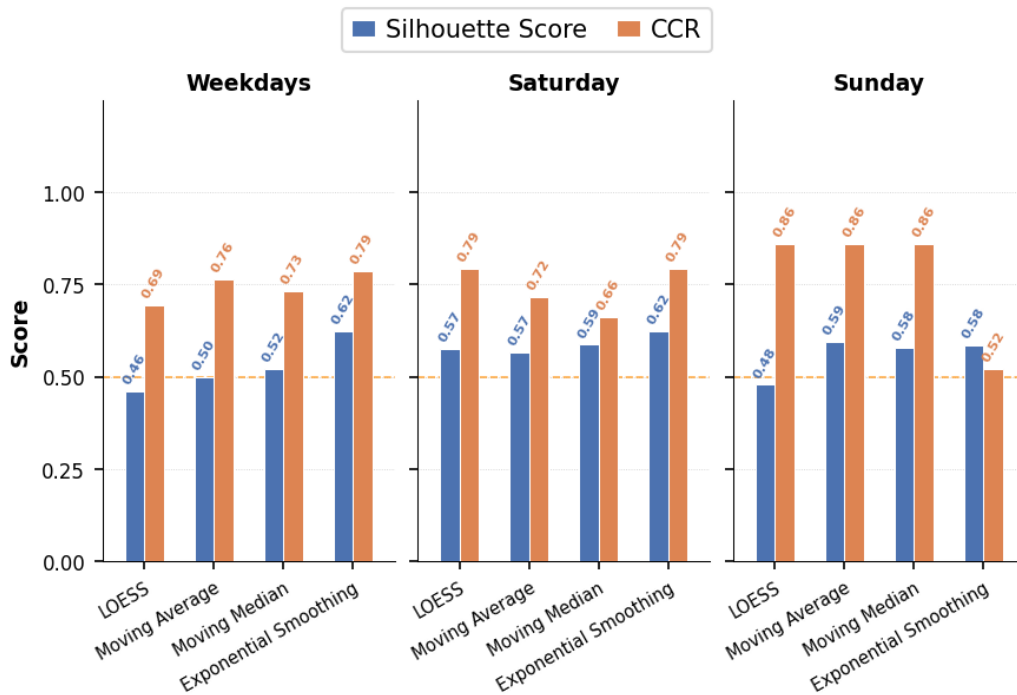
#### A.4. Additional figures of K-means clustering with 3 distance metrics

**Silhouette score and Correct Classification Rate (CCR):  
K-Means clustering with Correlation-based distance — Node 11**



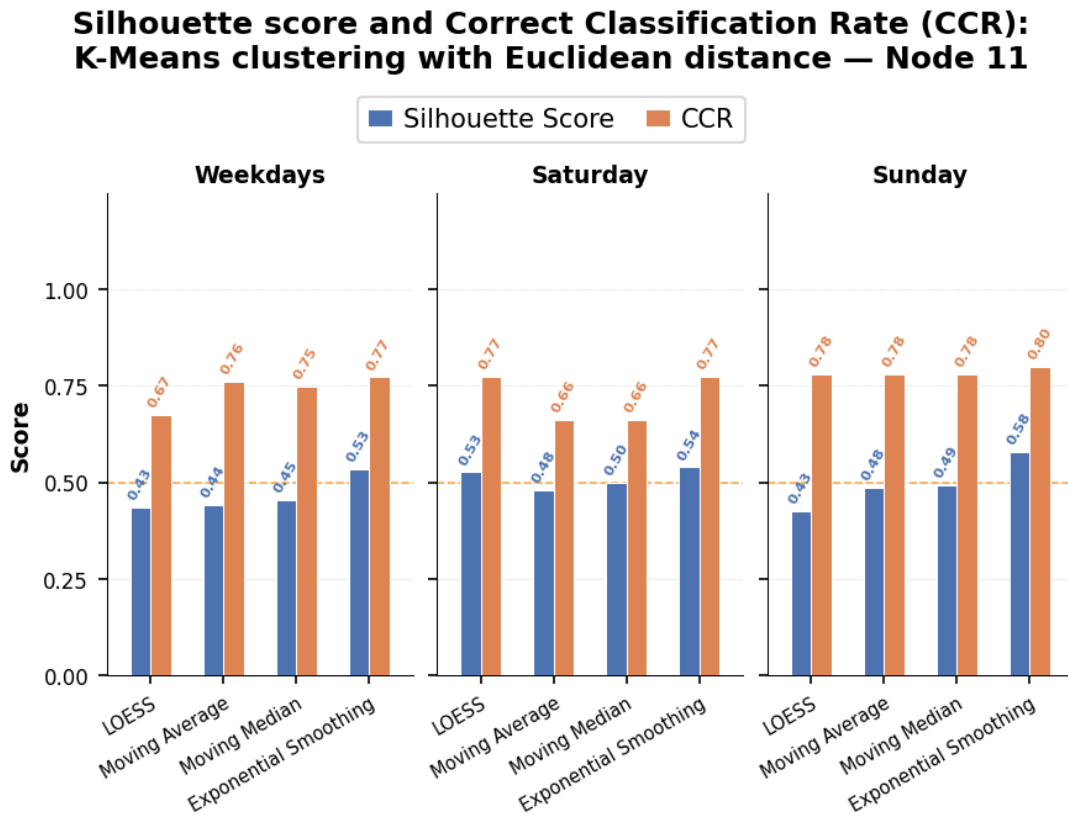
(a) Correlation-based distance.

**Silhouette score and Correct Classification Rate (CCR):  
K-Means clustering with DTW — Node 11**



(b) DTW distance.

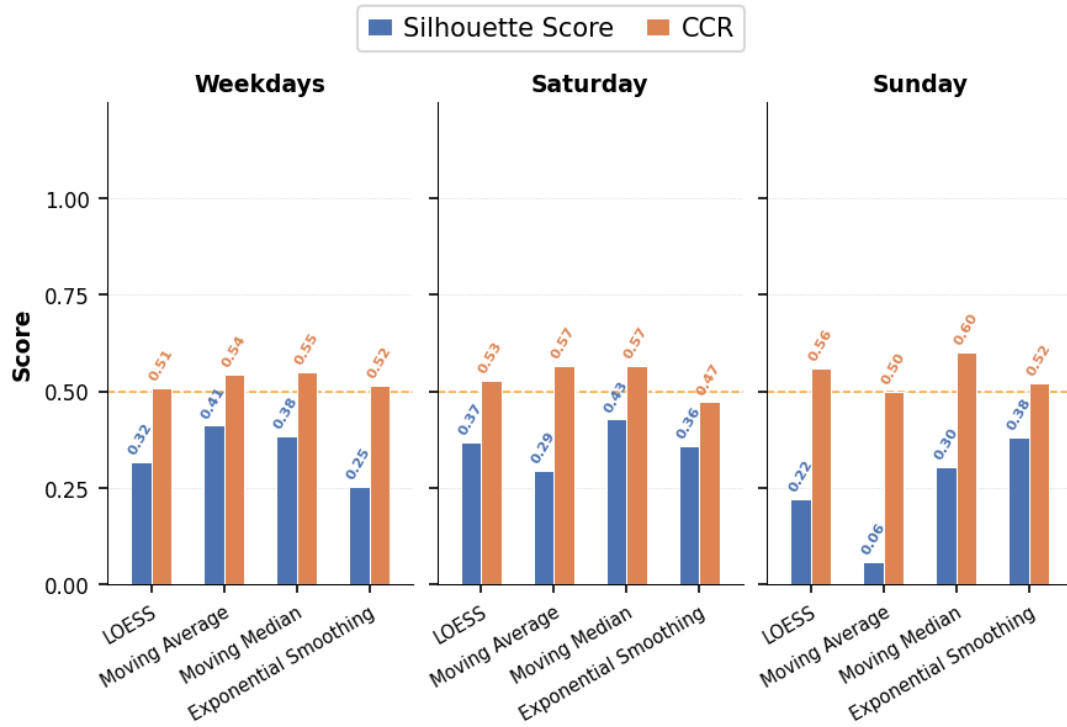
Figure 37: Silhouette score and CCR for K-Means clustering for Node 11.



(c) Euclidean distance.

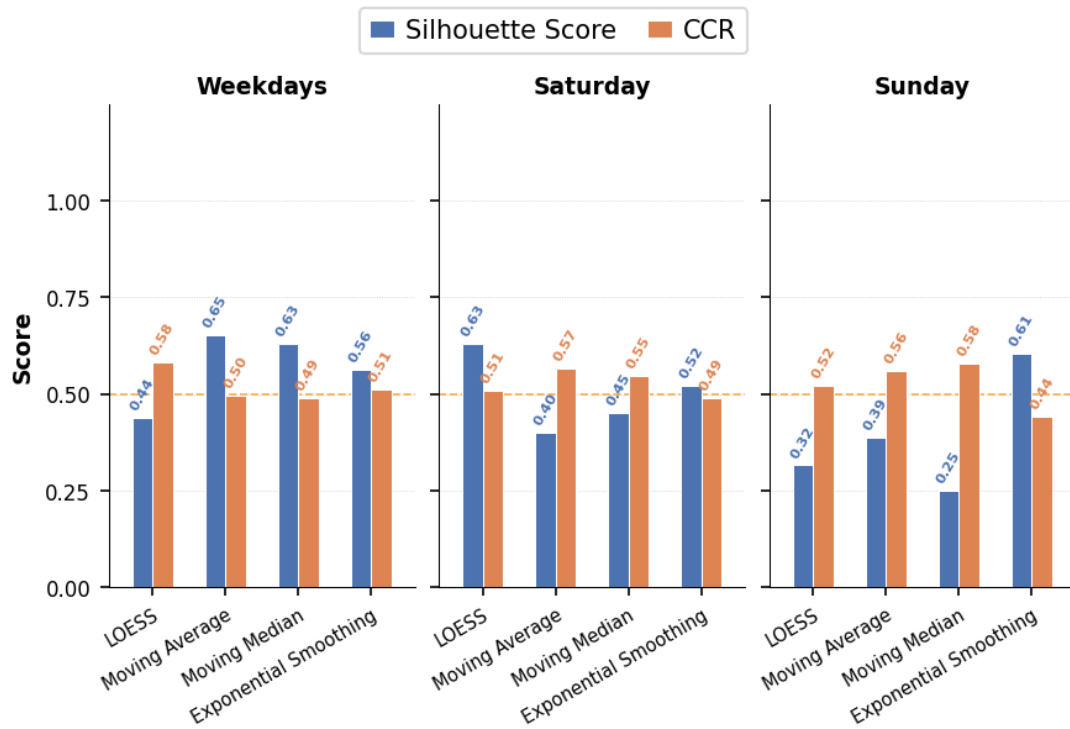
Figure 37: Silhouette score and CCR for K-Means clustering for Node 11 (continued).

**Silhouette score and Correct Classification Rate (CCR):  
K-Means clustering with Correlation-based distance — Node 12**



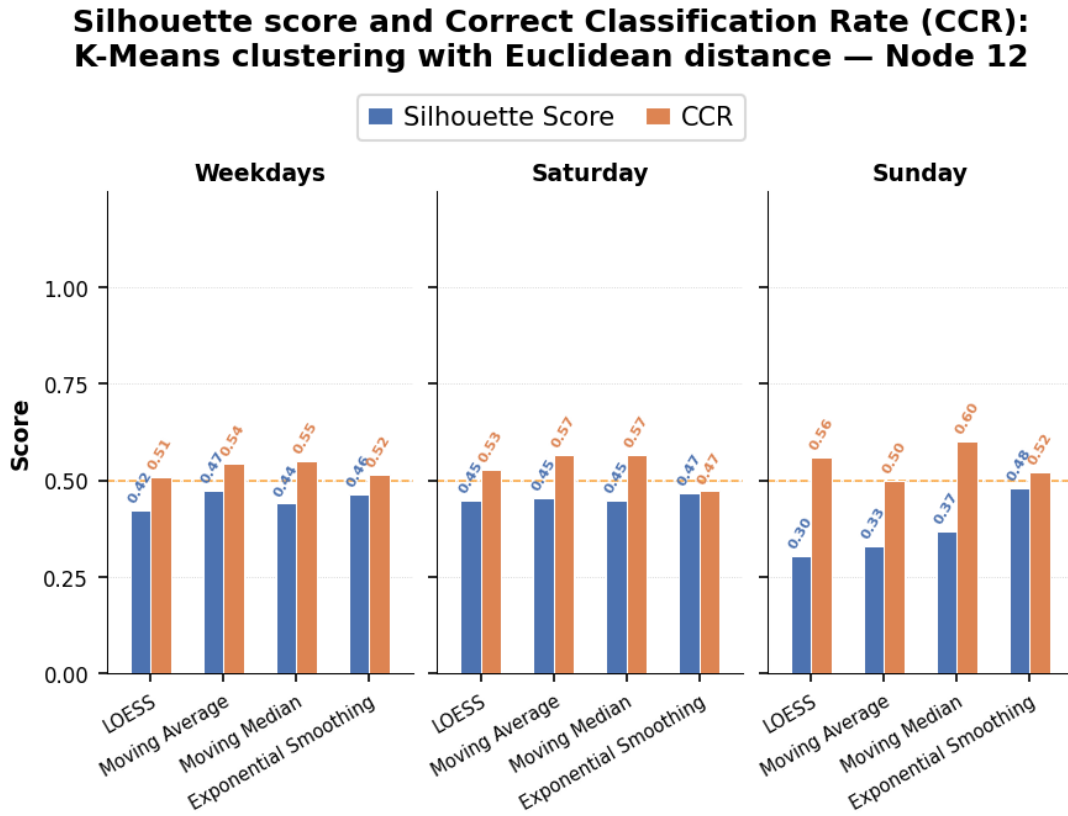
(a) Correlation-based distance.

**Silhouette score and Correct Classification Rate (CCR):  
K-Means clustering with DTW — Node 12**



(b) DTW distance.

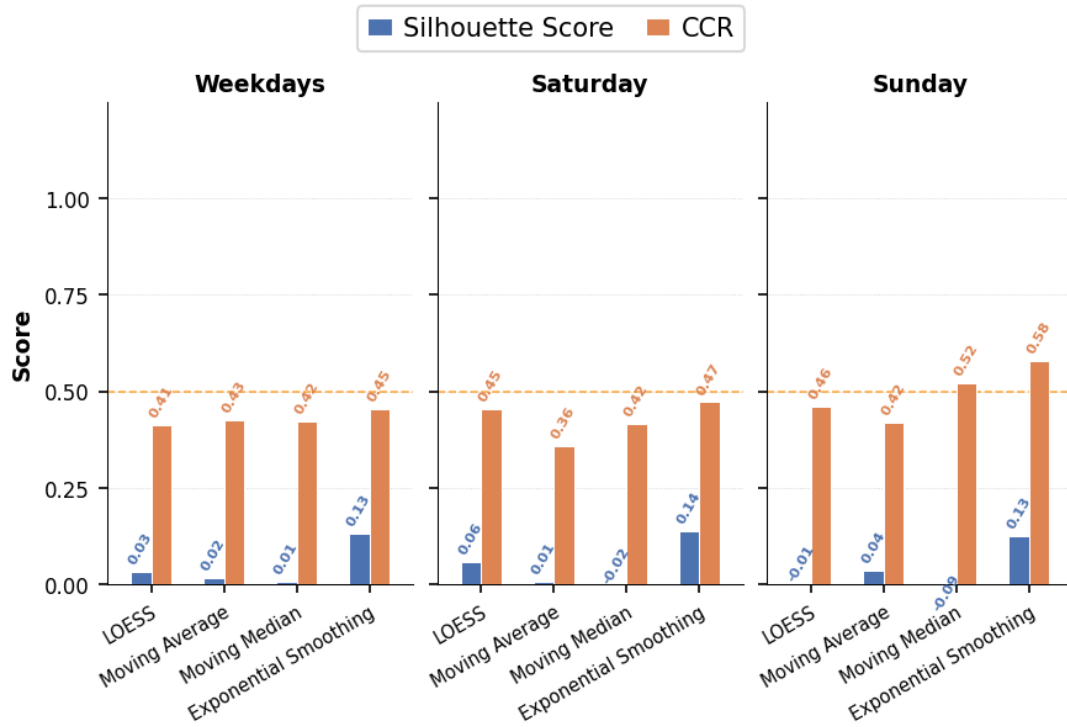
Figure 38: Silhouette score and CCR for K-Means clustering for Node 12.



(c) Euclidean distance.

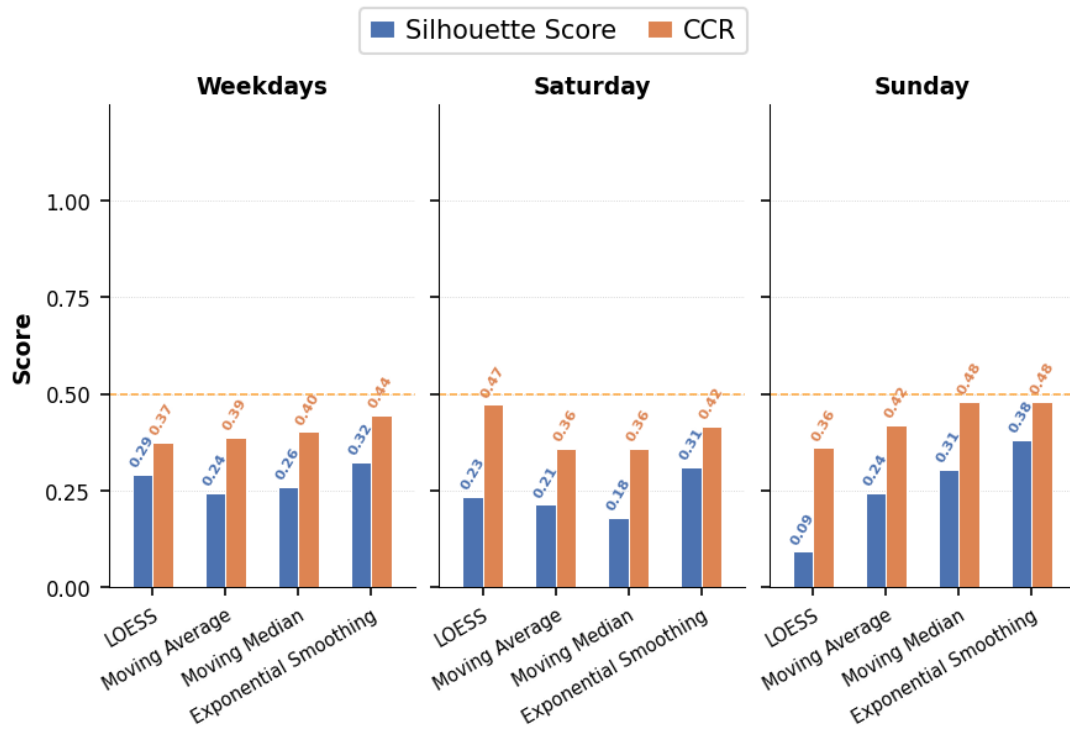
Figure 38: Silhouette score and CCR for K-Means clustering for Node 12 (continued).

**Silhouette score and Correct Classification Rate (CCR):  
K-Means clustering with Correlation-based distance — Node 13**



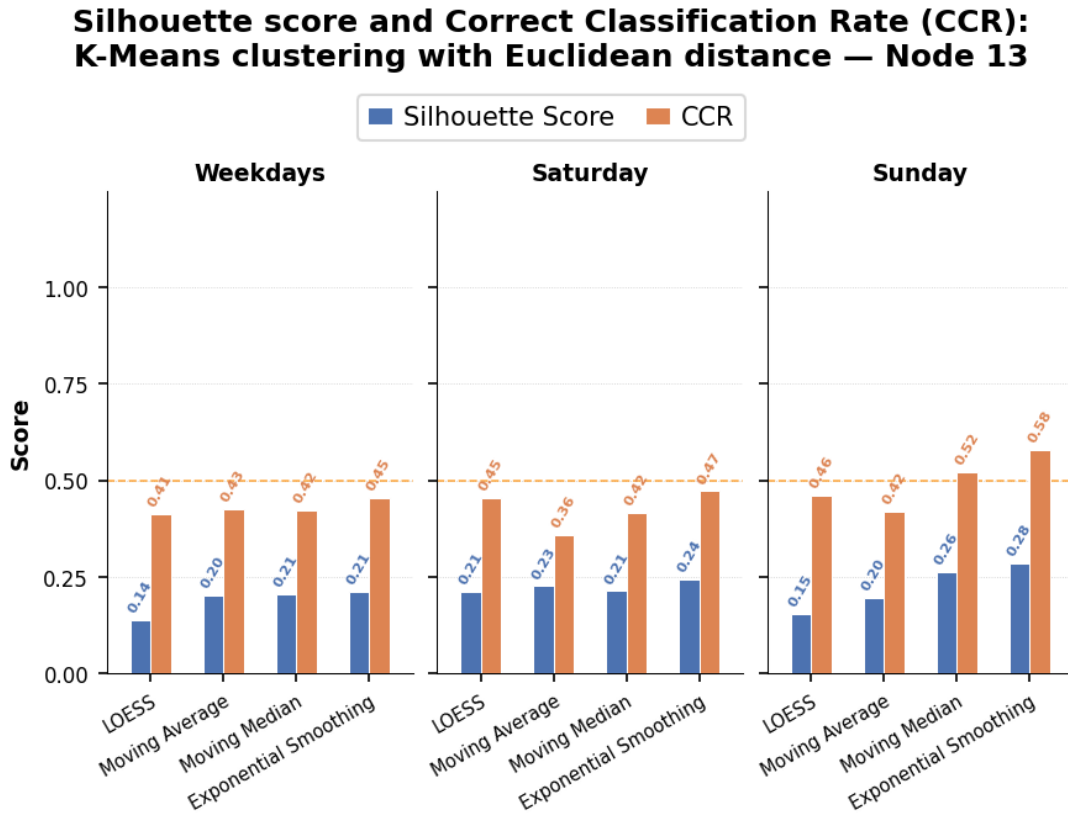
(a) Correlation-based distance.

**Silhouette score and Correct Classification Rate (CCR):  
K-Means clustering with DTW — Node 13**



(b) DTW distance.

Figure 39: Silhouette score and CCR for K-Means clustering for Node 13.



(c) Euclidean distance.

Figure 39: Silhouette score and CCR for K-Means clustering for Node 13 (continued).