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Further errata and comments to the first edition of

Iacus, S.M. (2008). Simulation and Inference for Stochastic Differential Equations. With R Examples. Springer, New York.

Chapter 1

P. 20/21: Figure 1.3 with the step function for $n = 10$ is misleading. It is caused by the fact that t is `t<-seq(0,T,length=100)` in all three cases in the R code. We think that `t<-seq(0,T,length=n)` leads to a better demonstration of the effect of $n = 10, 100, 1000$ for the random walk approach.

P. 20, last line of R code: `W <- as.numeric(W)/sqrt(n)` should be `W <- as.numeric(W)*sqrt(T/n)` to include also cases of $T \neq 1$ (Jakob Richter).

P.24, R code: `Wh` in the code is only a constant so that dividing by `Delta` is obviously a strongly decreasing function (Jakob Richter).

P.24, R code: To show the nondifferentiability of the Brownian motion with Karhunen-Loève expansion, a better code is the following:

```
function ()
{
set.seed(123)
phi<-function(i,t,T){
   (2*sqrt(2*T))/((2*i+1)*pi)*sin((2*i+1)*pi*t)/(2*T)
}
n<-10000
Z<-rnorm(n)
Delta<-seq(0.0005, 0.01,length=20)
```

```
W<-sum(Z*sapply(1:n, function(x) phi(x,0.5,T)))
Wh<-rep(0,20)
for(i in 1:20){
  Wh[i]<-sum(Z*sapply(1:n, function(x) phi(x,0.5+Delta[i],T)))
}
inc.ratio <- abs(Wh-W)/Delta
plot(Delta, inc.ratio,type="l")
}
```

P. 41, Line 2: $df(t, x)$ should be $df(t, X_t)$.

P. 41, Lines 15 to 17:

$$\begin{aligned} dX_t &= b_1(X_t)dt + \sigma(X_t)dW_t, \\ dX_t &= b_2(X_t)dt + \sigma(X_t)dW_t, \\ dX_t &= \sigma(X_t)dW_t \end{aligned}$$

should be

$$\begin{aligned} dX_t^1 &= b_1(X_t^1)dt + \sigma(X_t^1)dW_t, \\ dX_t^2 &= b_2(X_t^2)dt + \sigma(X_t^2)dW_t, \\ dX_t &= \sigma(X_t)dW_t. \end{aligned}$$

It would be also nice to use X_t^1, X_t^2 in Formulas (1.36) and (1.37).

P.45, Formulas (1.41) and (1.42): $m(t, x)$ and $v(t, x)$ should be $m(t, x_0)$ and $v(t, x_0)$, respectively.

P.45, R code:

```
> ito.sum <- c(0,sapply(2:N, function(x){
+    exp(-theta*(t[x]-t[x-1])) * (W[x]-W[x-1])} ))
```

should be

```
> ito.sum <- c(0,sapply(2:N, function(x){
+    sigma * exp(-theta*(-t[x-1])) * (W[x]-W[x-1])} ))
```

(Stefan Meinke). However, there is still something missing.

P. 47, Line -9:

$$X_t = \left(X_0 - \frac{\theta_1}{\theta_2}\right) e^{-\theta_2 t} + \theta_3 e^{-\theta_2 t} \int_0^t e^{\theta_2 u} \sqrt{X_u} dW_u$$

should be

$$X_t = \frac{\theta_1}{\theta_2} + \left(X_0 - \frac{\theta_1}{\theta_2}\right) e^{-\theta_2 t} + \theta_3 e^{-\theta_2 t} \int_0^t e^{\theta_2 u} \sqrt{X_u} dW_u$$

(Stefan Meinke).

P. 48, Line 1: $\left(\frac{u}{v}\right)^{q/2}$ should be $\left(\frac{v}{u}\right)^{q/2}$ (Philipp Probst).

P. 48, Variance of the CIR-Process:

$$v(t, x_0) = \left(x_0 - \frac{\theta_1}{\theta_2}\right) \frac{\theta_3^2(e^{-\theta_2 t} - e^{-2\theta_2 t})}{\theta_2} + \frac{\theta_1 \theta_3^2 (1 - e^{-2\theta_2 t})}{2\theta_2^2}$$

or

$$v(t, x_0) = x_0 \frac{\theta_3^2(e^{-\theta_2 t} - e^{-2\theta_2 t})}{\theta_2} + \frac{\theta_1 \theta_3^2 (1 - e^{-\theta_2 t})^2}{2\theta_2^2}$$

instead of

$$v(t, x_0) = x_0 \frac{\theta_3^2(e^{-\theta_2 t} - e^{-2\theta_2 t})}{\theta_2} + \frac{\theta_1 \theta_3^2 (1 - e^{-2\theta_2 t})}{2\theta_2^2}.$$

This can be shown with the Ito isometry.

P. 49, Table 1.4: The parameters θ_1, θ_2 of the CIR process are given as „any“ although they should satisfy $\theta_1, \theta_2 \in \mathbb{R}_+$ before on Page 47. It is also misleading that the parameter θ_2 is given as „any“ for the property of mean reverting of the CIR and OU processes (Stefan Meinke).

Chapter 3

P. 114, Formulas (3.10) and (3.11): x should be x_0.

P. 119, Line 10: $\left(\frac{u}{v}\right)^{q/2}$ should be $\left(\frac{v}{u}\right)^{q/2}$ (Philipp Probst).

P. 120, Line 9: $\log\left(\frac{u}{v}\right)$ should be $\log\left(\frac{v}{u}\right)$. It is correct in the R code. (Philipp Probst).

P. 125, Line 6: $\hat{\theta}^2_{1,n}\Delta$ should be $\hat{\theta}_{1,n}\Delta$. (Sermad Abbas)

P. 126, R code: The realisation of the Elerian transition density in `dcElerian` is not correct. Therefore Sermad Abbas developed the modification `dcElerian2`:

```
1 dcElerian2 <- function (x, t, x0 , t0 , theta , d, s, sx , log = FALSE ) {
2 ## values for calculating the density function
3 Dt <- t - t0
4 A <- s(t0 , x0 , theta ) * sx(t0 , x0 , theta ) * Dt/2
5 B <- -s(t0 , x0 , theta )/(2 * sx(t0 , x0 , theta )) + x0 + d(t0 ,
6 x0 , theta ) * Dt - A
7 z <- (x - B)/A
8 z[z < 0] <- NA # taking into account that z is not allowed to be negative
9 C <- 1/(( sx(t0 , x0 , theta )^2) * Dt)
10 tmp <- sqrt (C * z)
11
12 ##   case-by-case analysis for the argument of cosh
13 tmp2 <- numeric ( length (tmp ))
14 idx <- which ( abs(tmp) > 10)
15 tmp2 [idx ] <- tmp[idx] - log (2)
16 tmp2 [-idx ] <- log ( cosh (tmp[-idx ]))
17
18 ## log of the transition density
19 lik <- -log ( abs (A)) - 0.5 * log (2*pi) - 0.5 * log(z)
20 - (C + z)/2 + tmp2
21 if (!log)
22 lik <- exp (lik)
23 lik [is. infinite (lik )] <- NA
24
25 ## If sx == 0: Euler method
26 idx <- which (A == 0)
27 lik [idx] <- dcEuler (x, t, x0 , t0 , theta , d, s, log )[ idx ]
28 lik
29 }
```

The first difference to `dcElerian` is in Line 9 where `s(t0 , x0 , theta )` is replaced by `sx(t0 , x0 , theta )`. The next deviation can be found in Line 14. In `dcElerian`, the value 500 is used. However, this can produce error messages in the optimization so that the value was reduced to 10. We assumed that cosh is growing to fast which can produce numerical problems. Furthermore $\log(2\pi)$ is replaced by $\frac{1}{2}\log(2\pi)$ in Line 19. This has no influence on the optimization. But now, it is ensured that correct values of the transition density are obtained. With the modified Elerian method `dcElerian`, Sermad Abbas was able to achieve very good simulation results for the Black-Scholes-Merton model and the Cox-Ingersoll-Ross model without using the Lamperti transform. In the optimization he used the function `mle` with the option `method = ''L-BFGS-B''`, with lower bound 0.1 and upper bound 1.5 for the parameters.

P. 158, Line 7: $p_\theta(\Delta, X_{i-1}|X_i)$ should be $p_\theta(\Delta, X_i|X_{i-1})$ (Julia Funk).

P. 158, Formula (3.37):

$$\sum_{i=1}^{n} \left\{ b(X_i, \theta) h'(X_i) + \frac{1}{2}\sigma^2(X_i, \theta) h''(X_i) \right\}$$

should be

$$\sum_{i=1}^{n} \left\{ b(X_{i-1}, \theta) h'(X_{i-1}) + \frac{1}{2}\sigma^2(X_{i-1}, \theta) h''(X_{i-1}) \right\}.$$

P. 159, Line -7: $(2\theta X_{i-1}^2 - 1)$ should be $(1 - 2\theta X_{i-1}^2)$ (Julia Funk).

P. 159, Line -4:

$$\frac{1 + e^{2\theta_0 \Delta}}{1 - e^{2\theta_0 \Delta}}$$

should be

$$\frac{1 + e^{2\theta_0 \Delta}}{e^{2\theta_0 \Delta} - 1} \text{ or } \frac{1 + e^{-2\theta_0 \Delta}}{1 - e^{-2\theta_0 \Delta}}$$

(Julia Funk).

P. 160, Formula (3.41): The term $8x^2\theta^2\Delta + 8yx\theta^2\Delta e^{3\theta\Delta} - 8yx\theta^2\Delta e^{\theta\Delta} - 8x^2\theta^2\Delta e^{2\theta\Delta}$ is missing (Julia Funk).

P. 160: Formula (3.41) means $f(x, y, \theta) = 0$. But correct is only $F(X^{obs}, \theta) = \sum_{i=1}^{n} f(X_{i-1}, X_i, \theta) = 0$ (Julia Funk).

P. 162, Line 1: $\Delta = 0.5$ should be $\Delta = 5.0$ (Julia Funk).

P. 184, Line 13: $g_n(\theta)^T g_n(\theta)$ instead of $g_n(\theta) g_n(\theta)^T$ (Philipp Hallmeier).

P. 185, Line 1: $J = n\ g_n(\hat\theta)^T W g_n(\hat\theta)$ instead of $J = g_n(\hat\theta) W g_n(\hat\theta)$ (Philipp Hallmeier).

P. 186, Line -1: $\Delta = 0.01$ should be $\Delta = 0.1$, and 5000 should be 2500 (Sebastian Szugat).

P. 187, Line 3:

$$\mathrm{Var}\{X_{i+1}|X_i = y\} = \left(y - \frac{\theta_1}{\theta_2}\right) \frac{\theta_3^2(e^{-\theta_2 t} - e^{-2\theta_2 t})}{\theta_2} + \frac{\theta_1\theta_3^2(1 - e^{-2\theta_2 t})}{2\theta_2^2}$$

or

$$\mathrm{Var}\{X_{i+1}|X_i = y\} = y\,\frac{\theta_3^2(e^{-\theta_2 t} - e^{-2\theta_2 t})}{\theta_2} + \frac{\theta_1\theta_3^2(1 - e^{-\theta_2 t})^2}{2\theta_2^2}$$

instead of

$$\mathrm{Var}\{X_{i+1}|X_i = y\} = y\,\frac{\theta_3^2(e^{-\theta_2 t} - e^{-2\theta_2 t})}{\theta_2} + \frac{\theta_1\theta_3^2(1 - e^{-2\theta_2 t})}{2\theta_2^2}.$$

(Philipp Hallmeier, second form).

P. 187, Line 4: $(\alpha, \beta, \sigma) = (\theta_1, -\theta_2, \theta_3)$ instead of $(\alpha, \beta, \sigma) = (\theta_1, \theta_2, \theta_3)$.