DD-Plots

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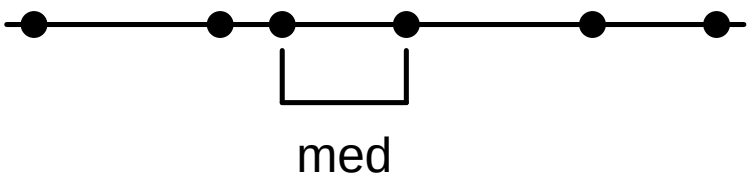
What is data depth?

Median:



1. Data depth as generalization of the median

What is data depth?

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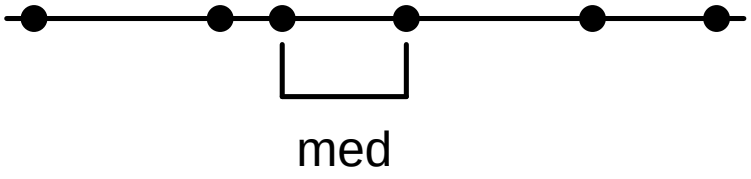
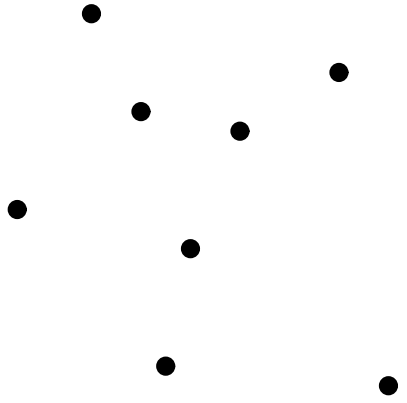
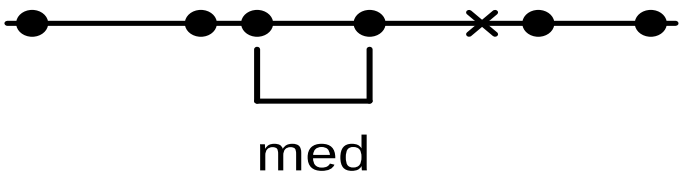


Generalization for multivariate data?



Data depth of μ in $z_* = (z_1, \ldots, z_N)$ (univariate):

$$d(\mu, z_*) = \frac{1}{N} \min \{\sharp\{n;\ z_n \leq \mu\}, \sharp\{n;\ z_n \geq \mu\}\}$$



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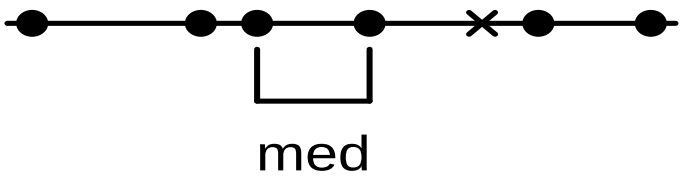

Here $d(\mu, z_*) = \frac{2}{6}$

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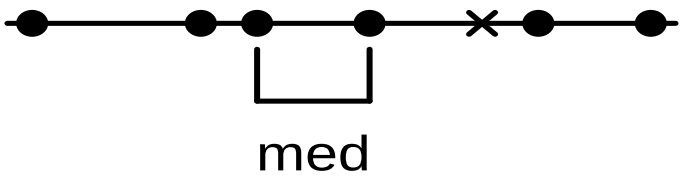
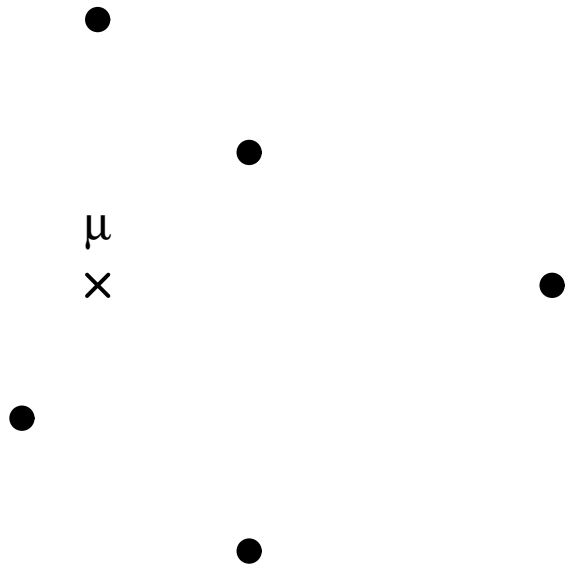


Here $d(\mu, z_*) = \frac{2}{6}$

Median: all μ with maximum data depth, i.e. all μ with $d(\mu, z_*) = \frac{1}{2}$

Data depth of μ in $z_* = (z_1, \ldots, z_N)$ (multivariate):
Half space depth of Tukey 1975

$$d_H(\mu, z_*) = \frac{1}{N} \min_H \sharp\{n;\ z_n \text{ lies in a half space } H \text{ containing } \mu\}$$



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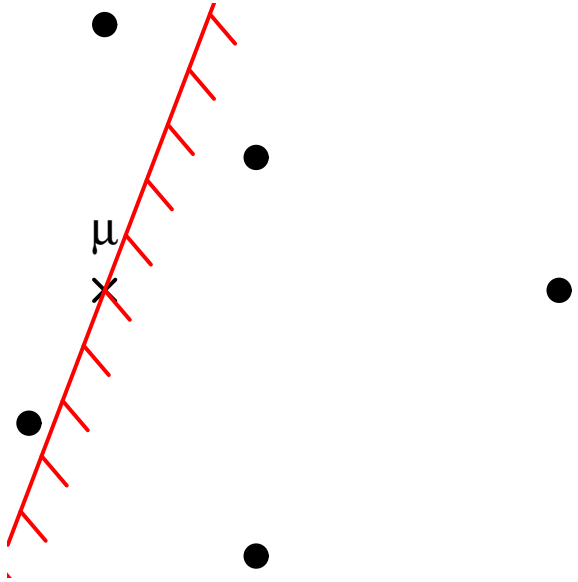

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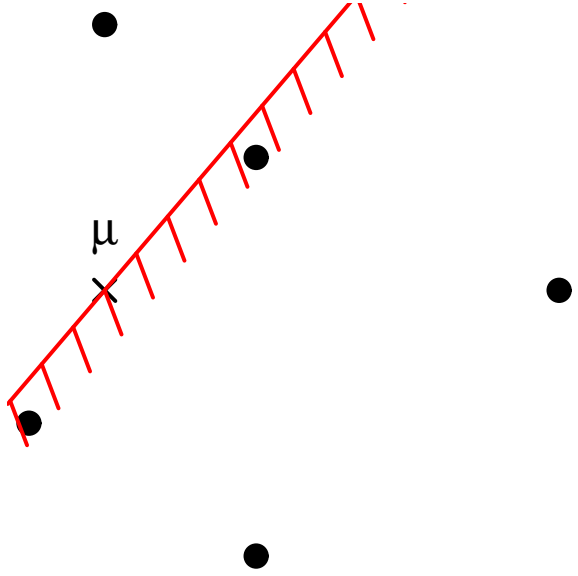

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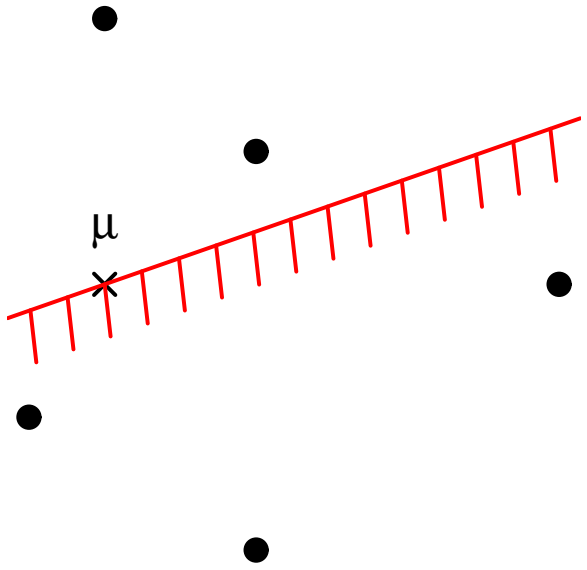

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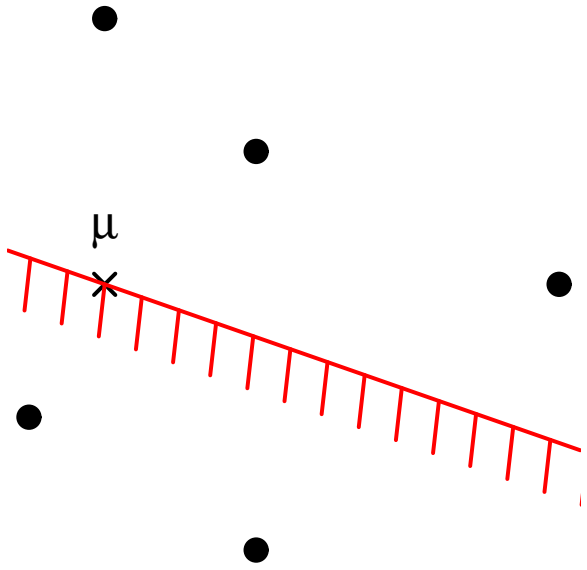

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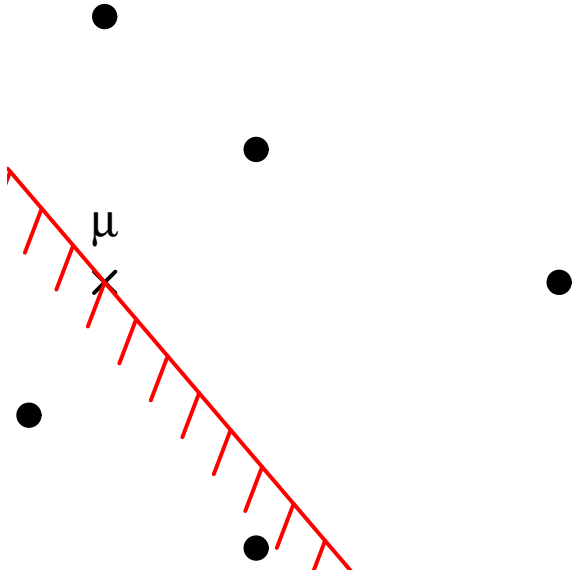

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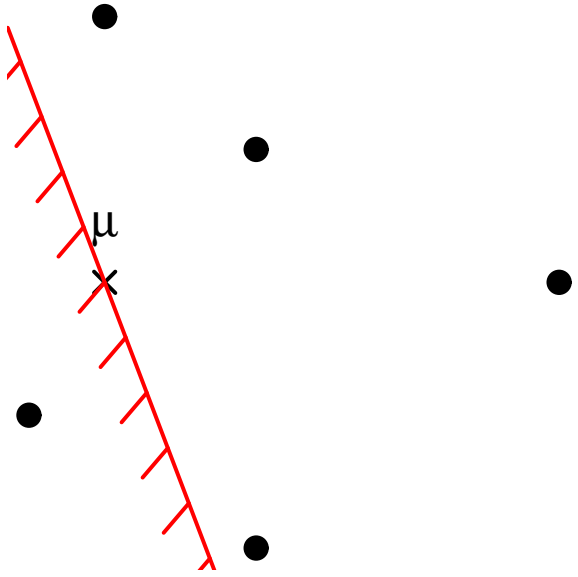
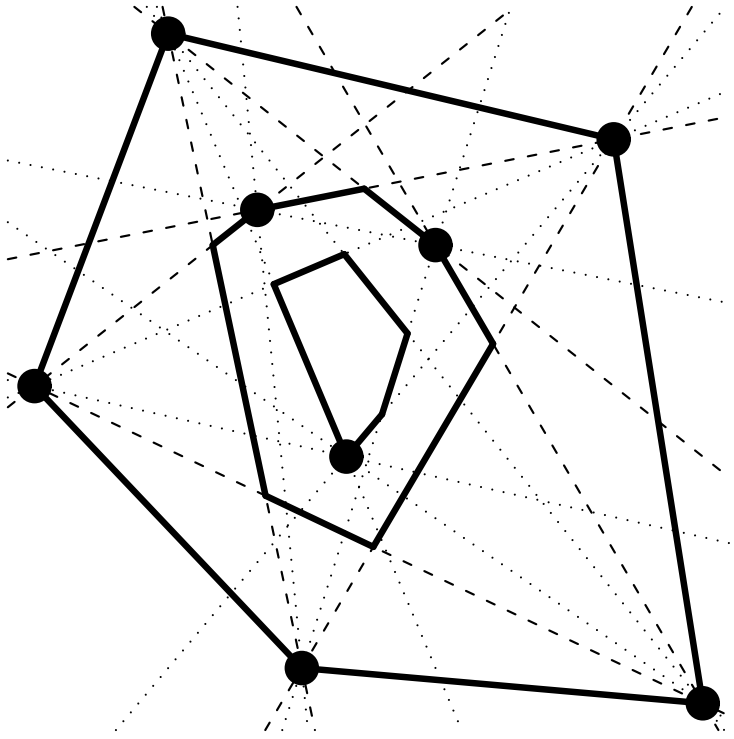
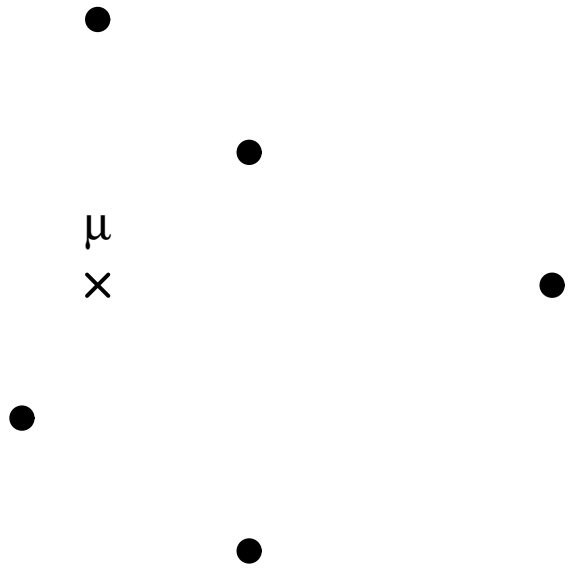


Here: $d_H(\mu, z_*) = \frac{1}{5} \cdot 1$

Half space median / Tukey median:
all μ with maximum half space depth



Simplicial depth $d_S(\mu, z_*)$ of a location parameter μ (Liu 1988,1990):
The relative number of simplicies (triangles) containing μ



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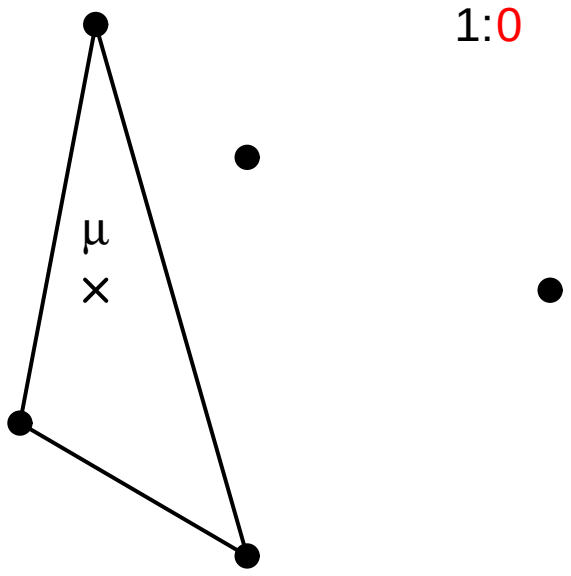

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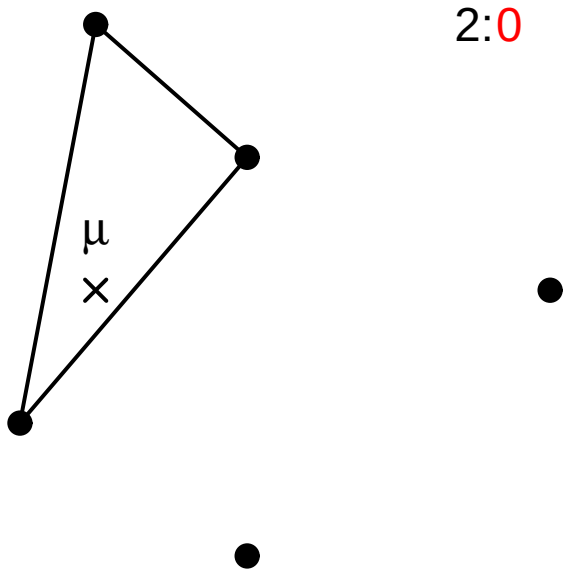

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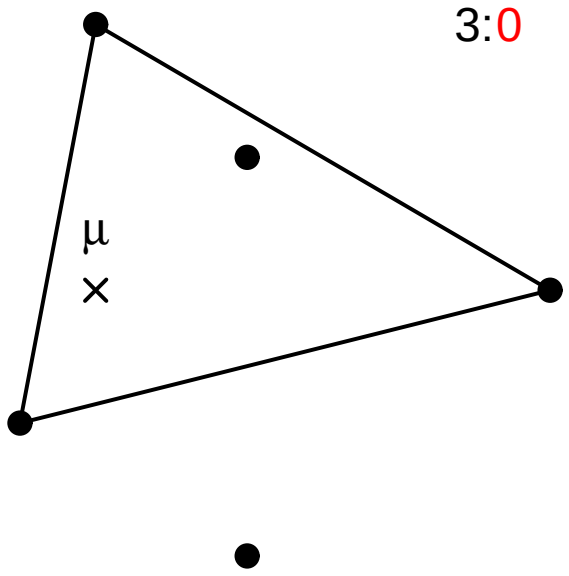

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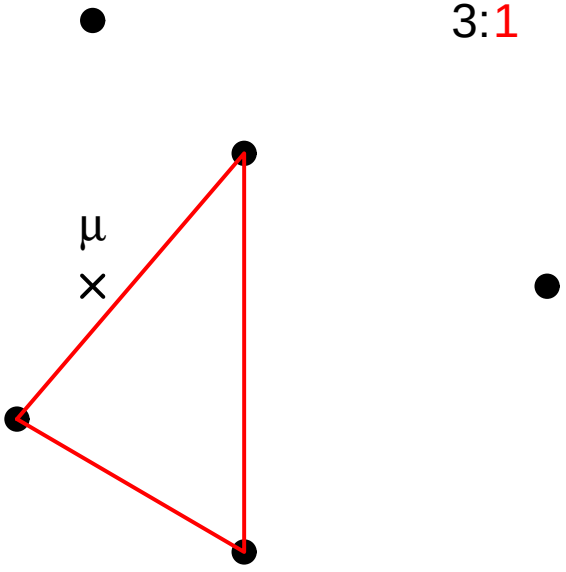

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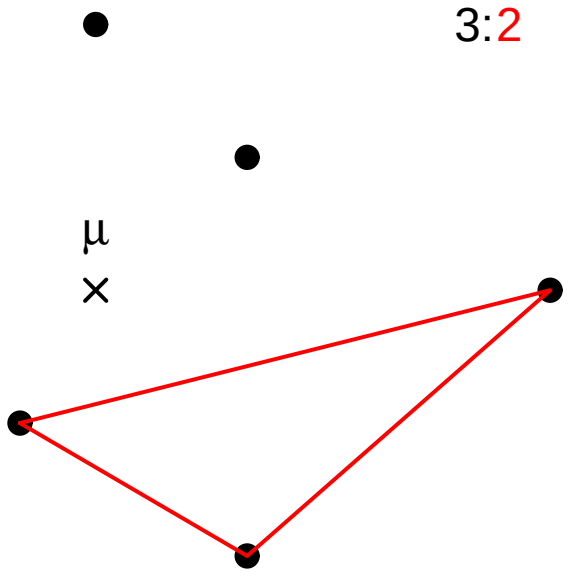

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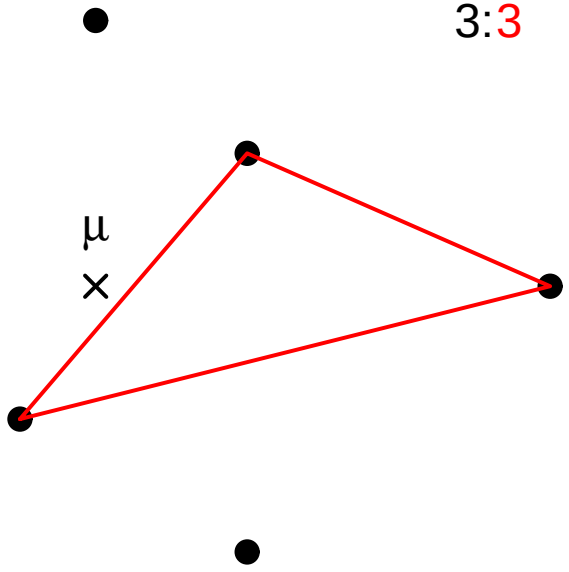

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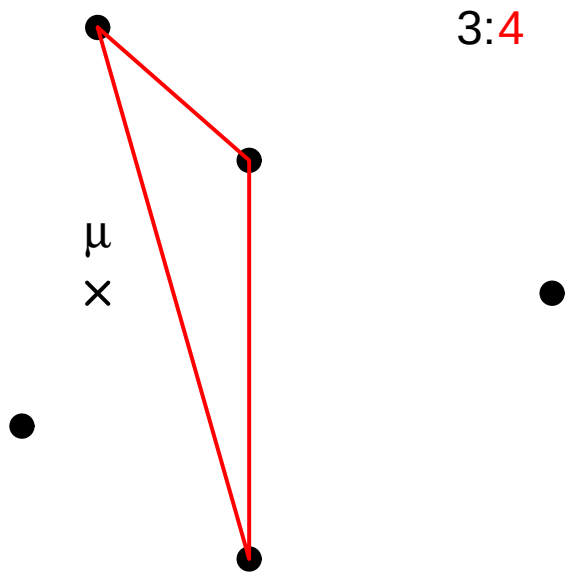

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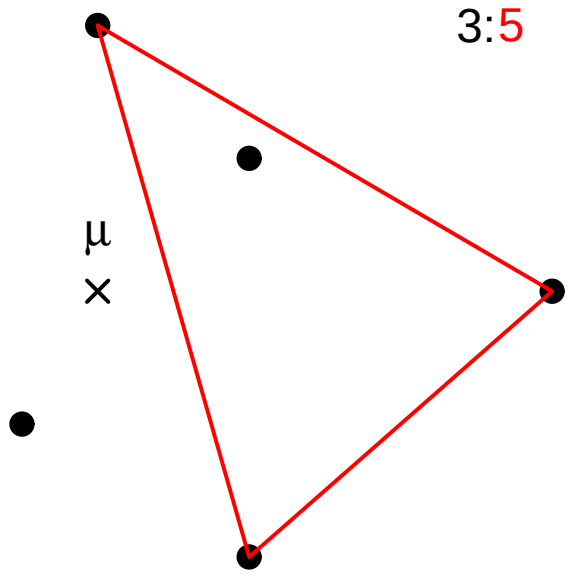

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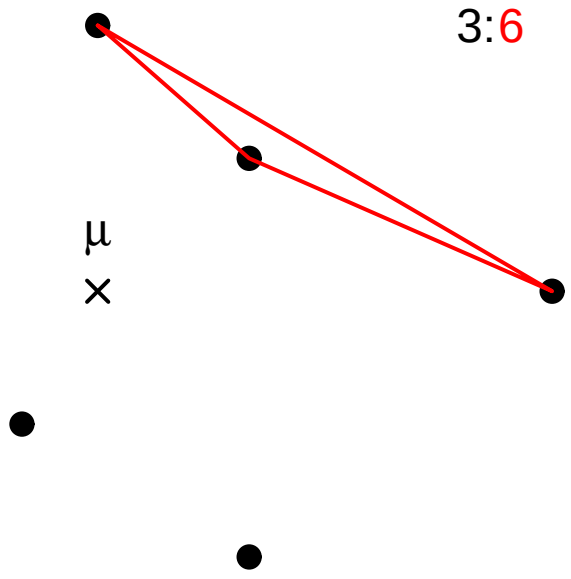

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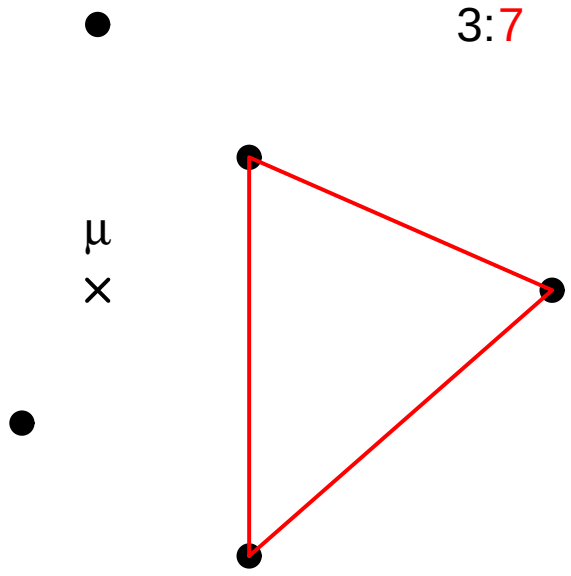

Here $d_S(\mu, z_*) = \frac{1}{\binom{5}{3}} \cdot 3 = \frac{1}{10} \cdot 3$

$= \frac{1}{\binom{N}{3}} \cdot \sharp\{\{n_1, n_2, n_3\};\ \mu \in$ simplex spanned by $z_{n_1}, z_{n_2}, z_{n_3}\}$

Group 0: $(z_1, \ldots, z_N)$

Group 1: $(z_{N+1}, \ldots, z_{N+M})$

$z_n \in I\!R^K$ for $n = 1, \ldots, N + M$

Depth-Depth-Plot (Li et al. 2012):

Plot of $(d(z_n, (z_1, \ldots, z_N)),\ d(z_n, (z_{N+1}, \ldots, z_{N+M})))$

for $n = 1, \ldots, N + M$

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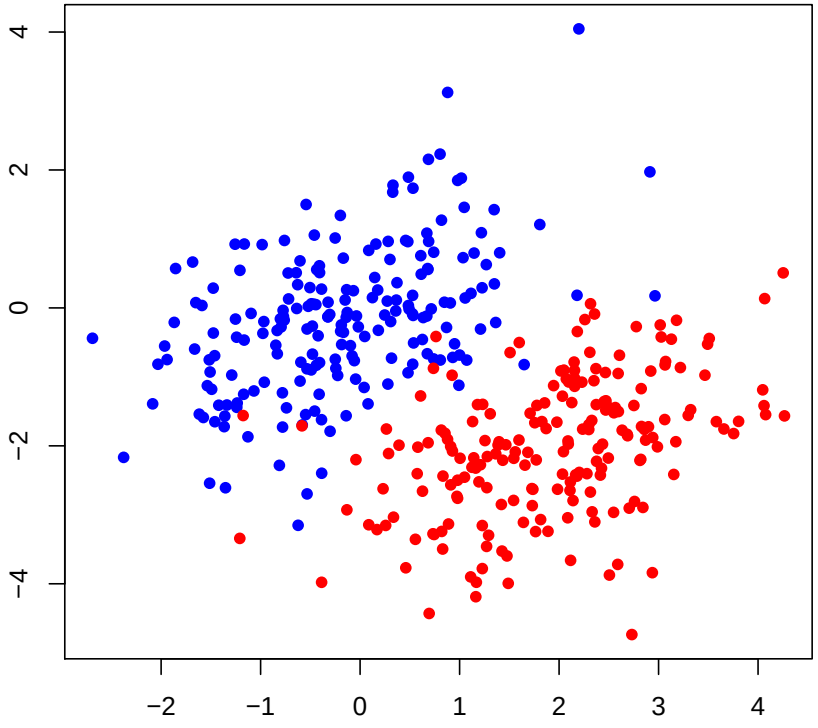
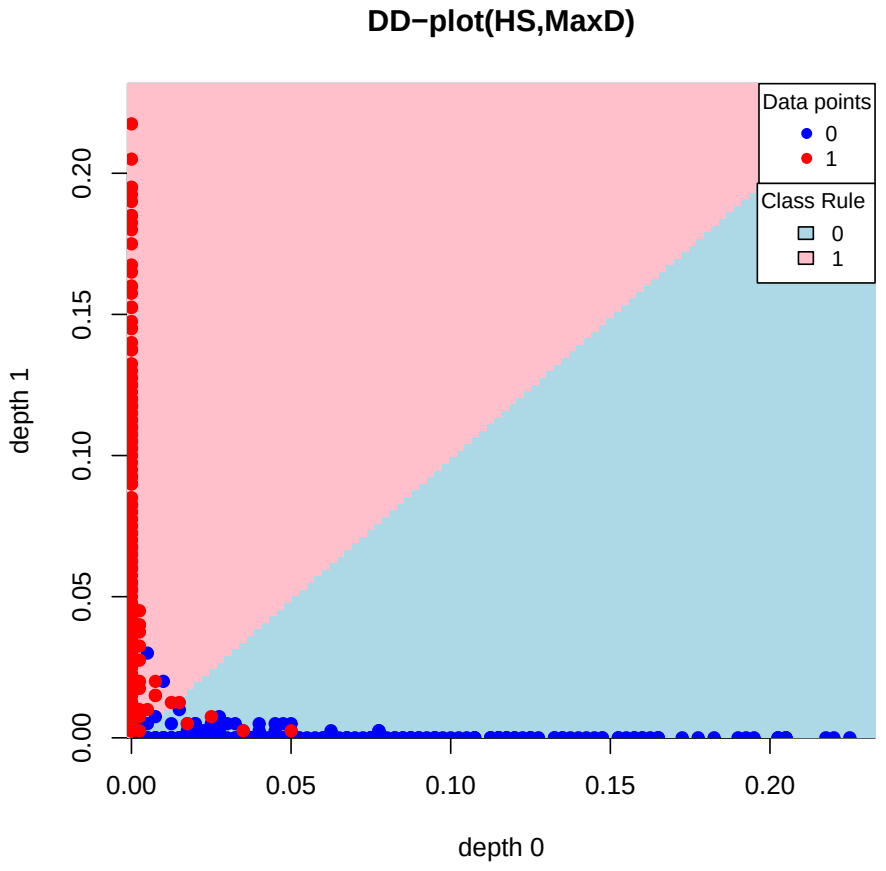
Scatter-Plot and Depth-Depth-Plot for $K = 2$: Almost separated groups





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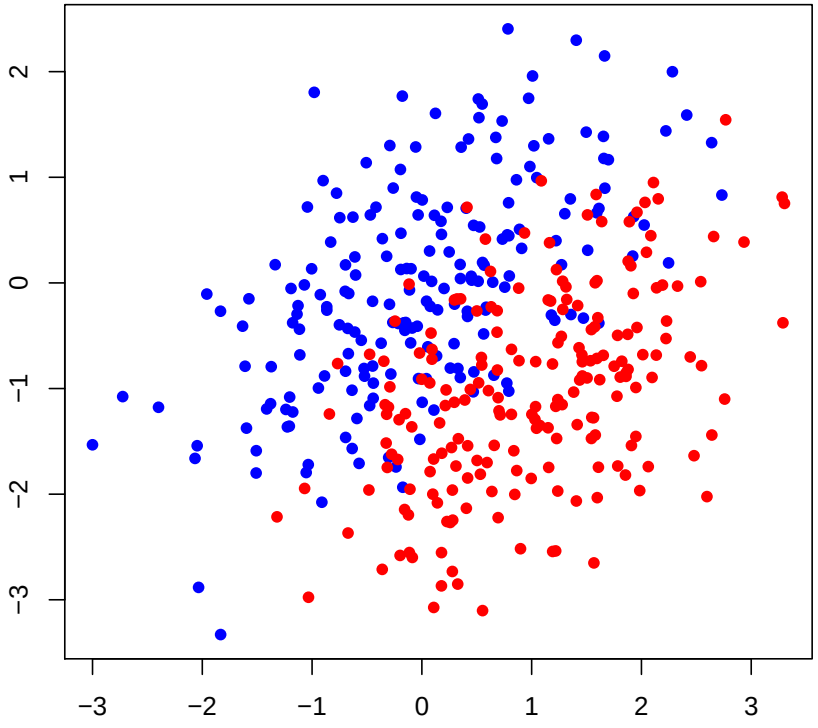
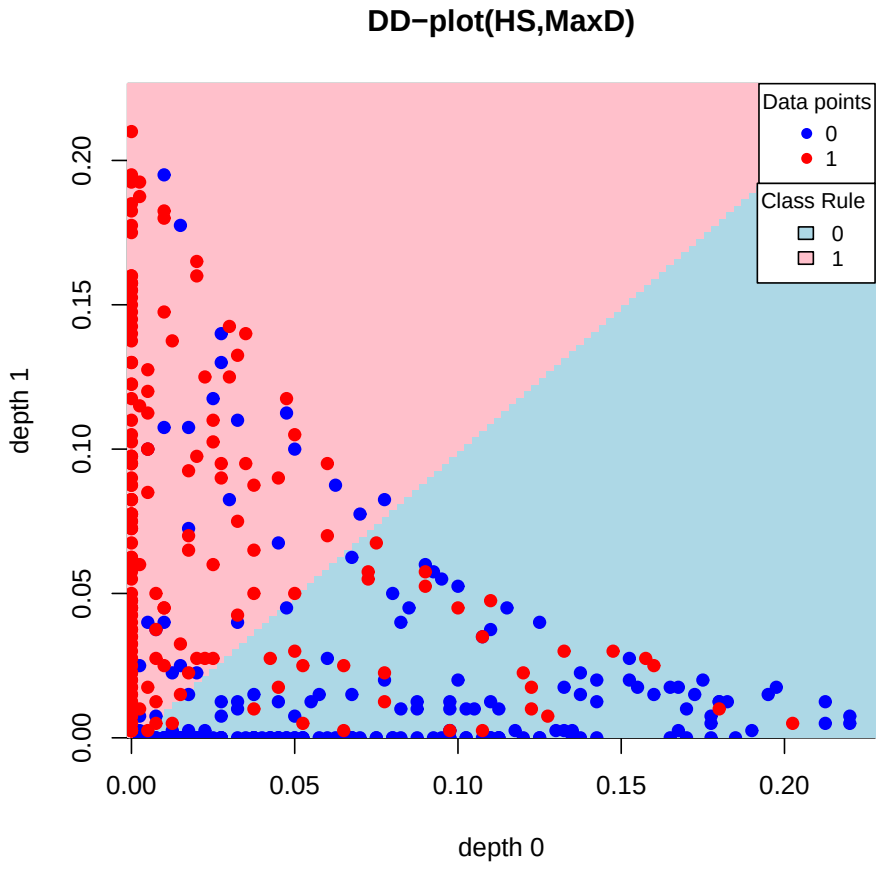
Scatter-Plot and Depth-Depth-Plot for $K = 2$: Overlapping groups





Classifying a new observation z_0:

$$d(z_0, (z_1, \ldots, z_N)) > d(z_0, (z_{N+1}, \ldots, z_{N+M})) \longrightarrow \text{Group 0}$$

$$d(z_0, (z_1, \ldots, z_N)) < d(z_0, (z_{N+1}, \ldots, z_{N+M})) \longrightarrow \text{Group 1}$$

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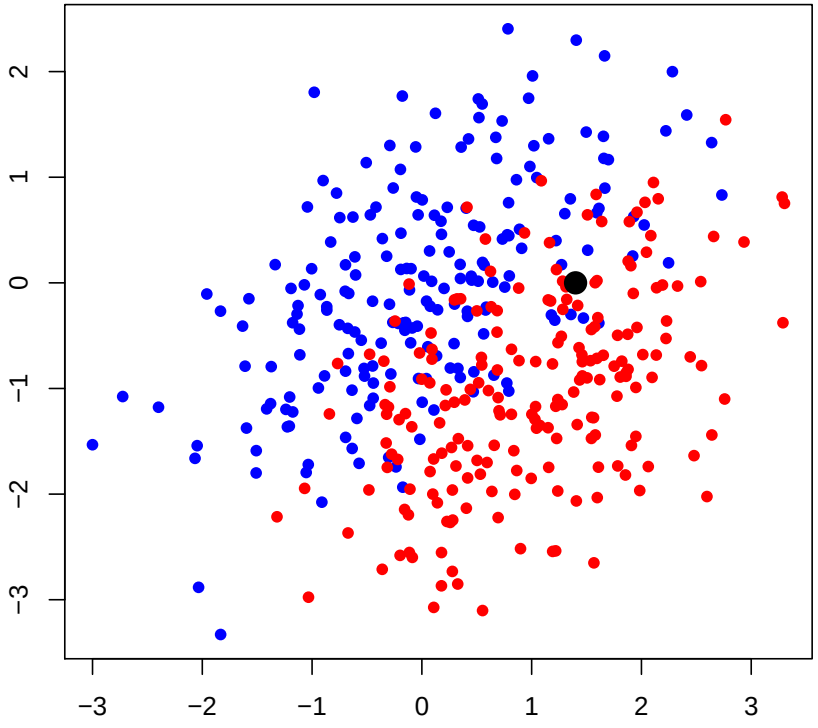
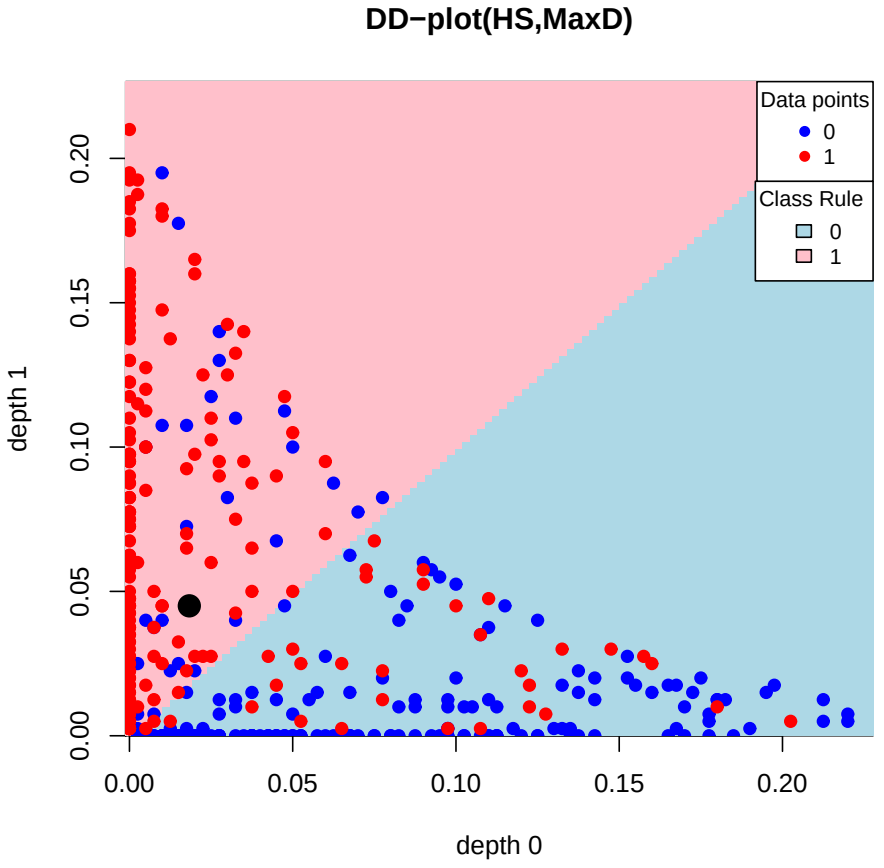
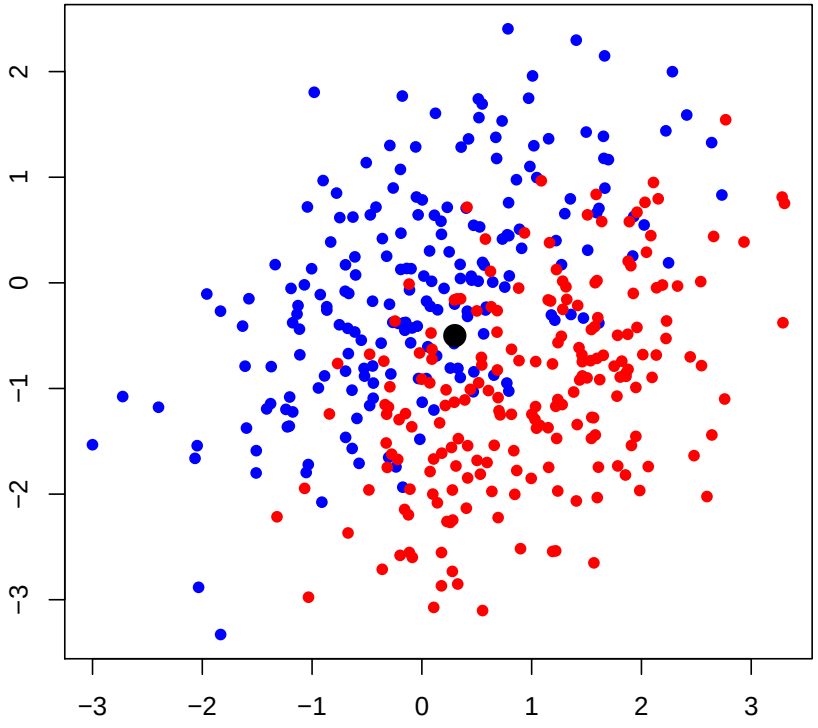
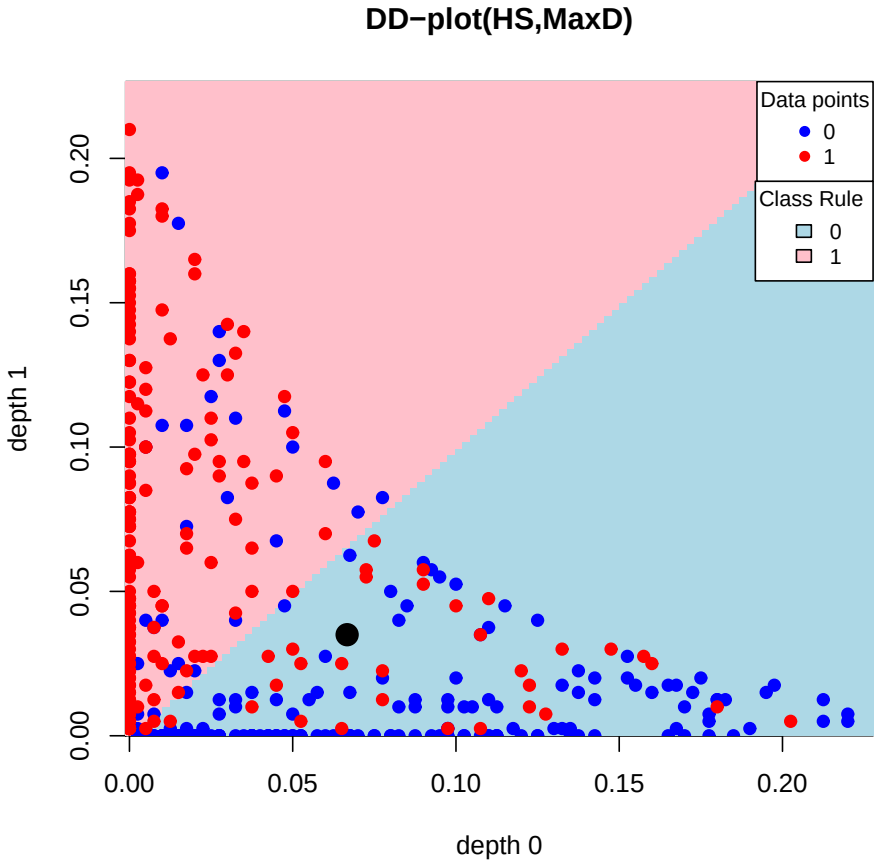
Classifying a new observation: new observation $\longrightarrow$ group 1



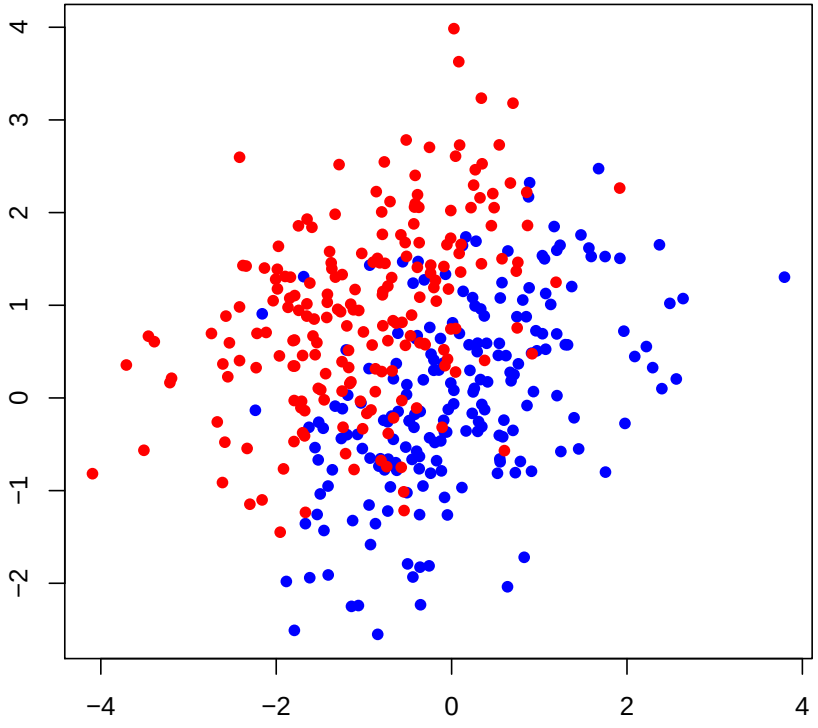


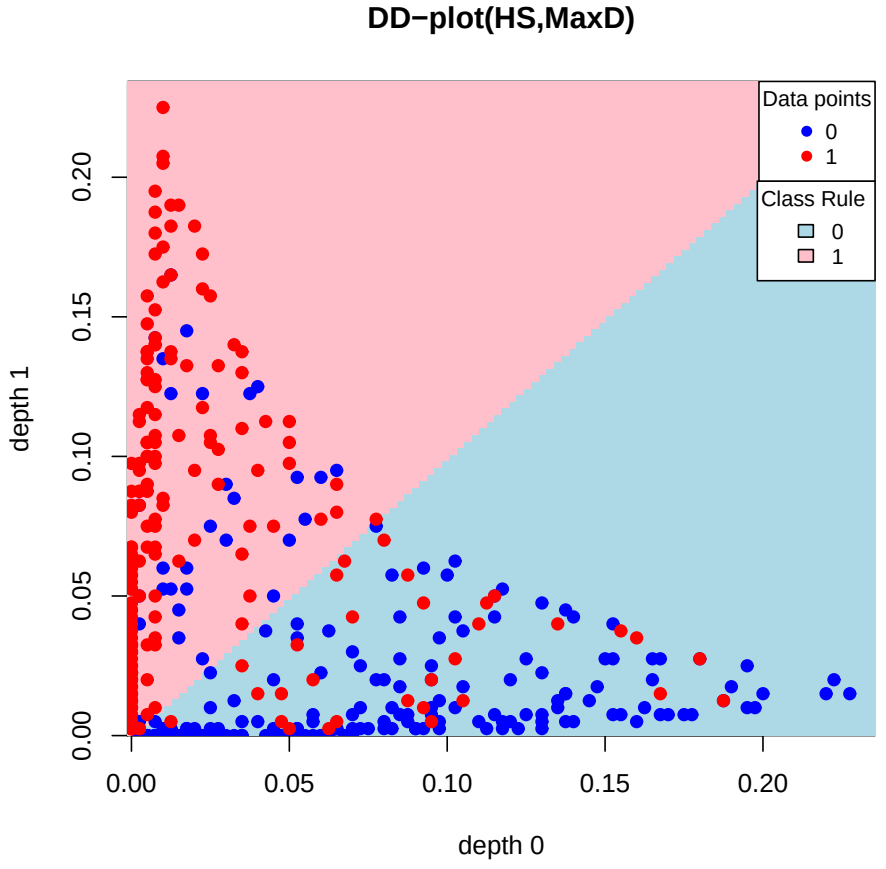
Classifying a new observation: new observation $\longrightarrow$ group 0



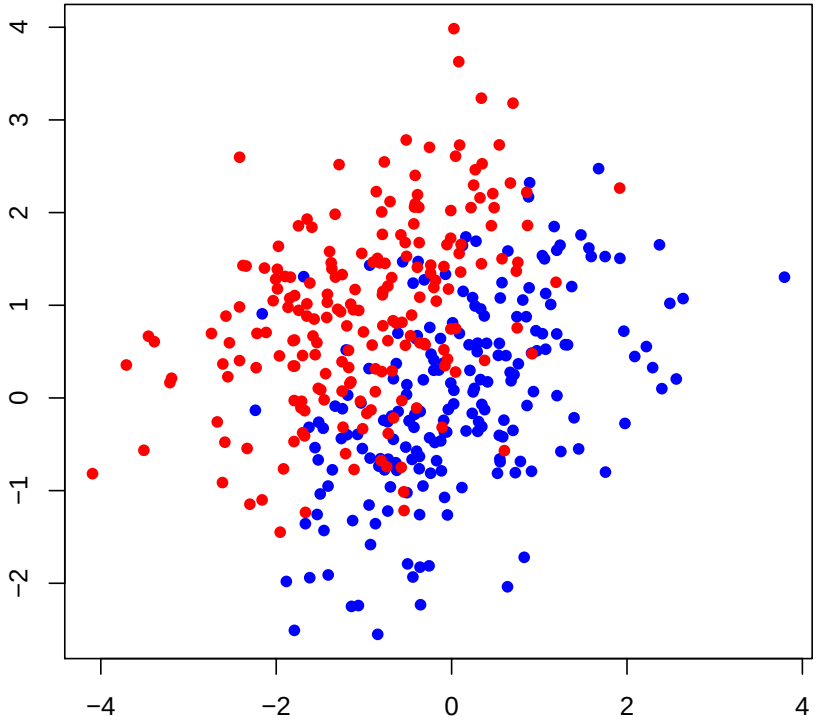


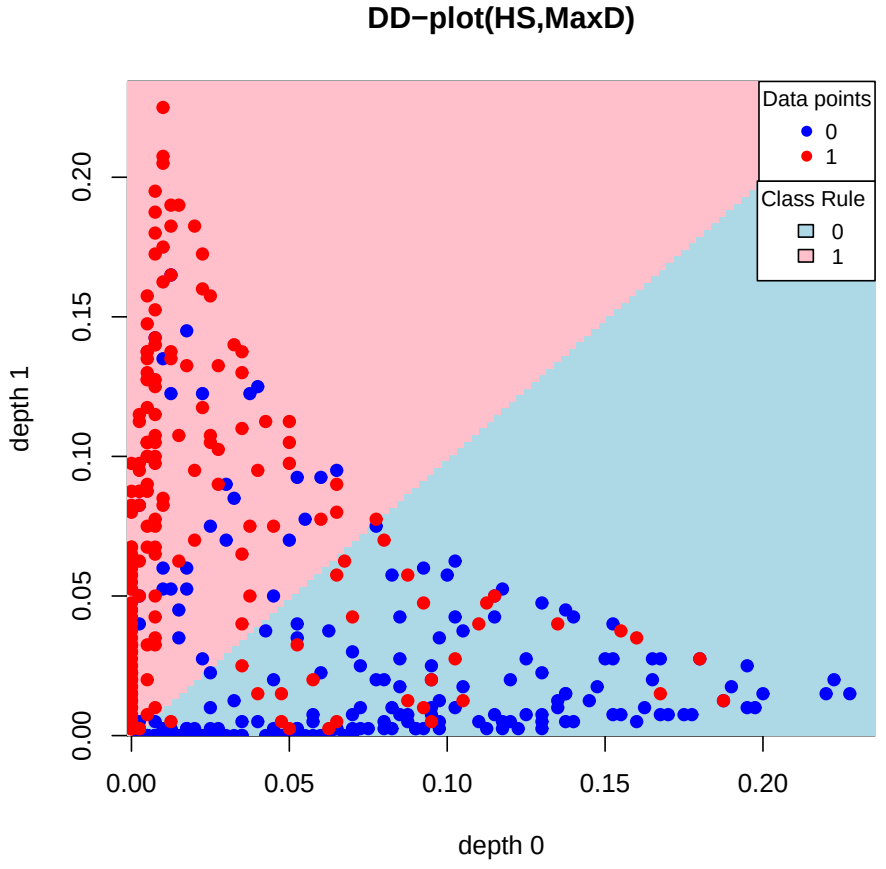
Prediction error: DD-HS=0.186



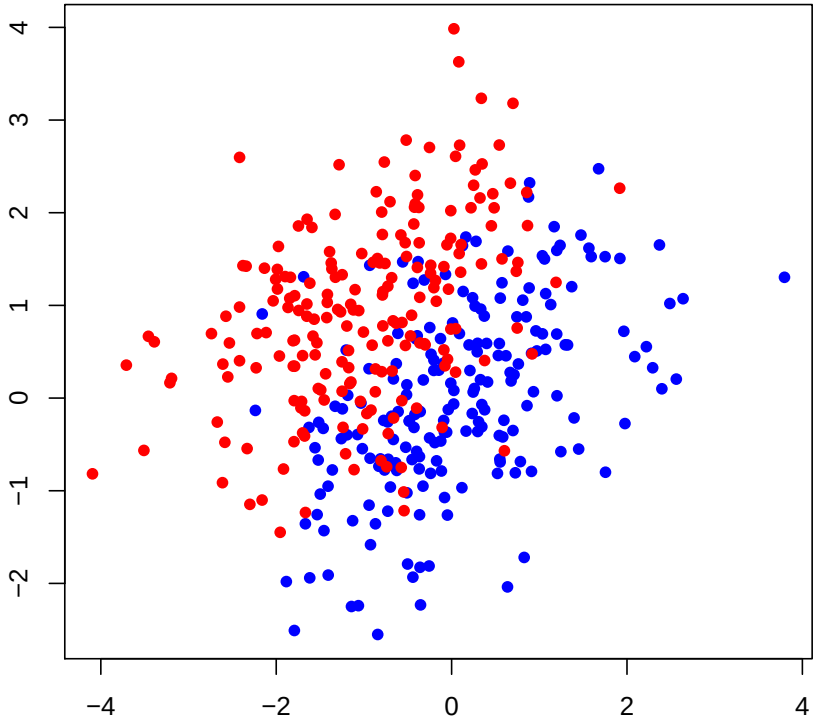


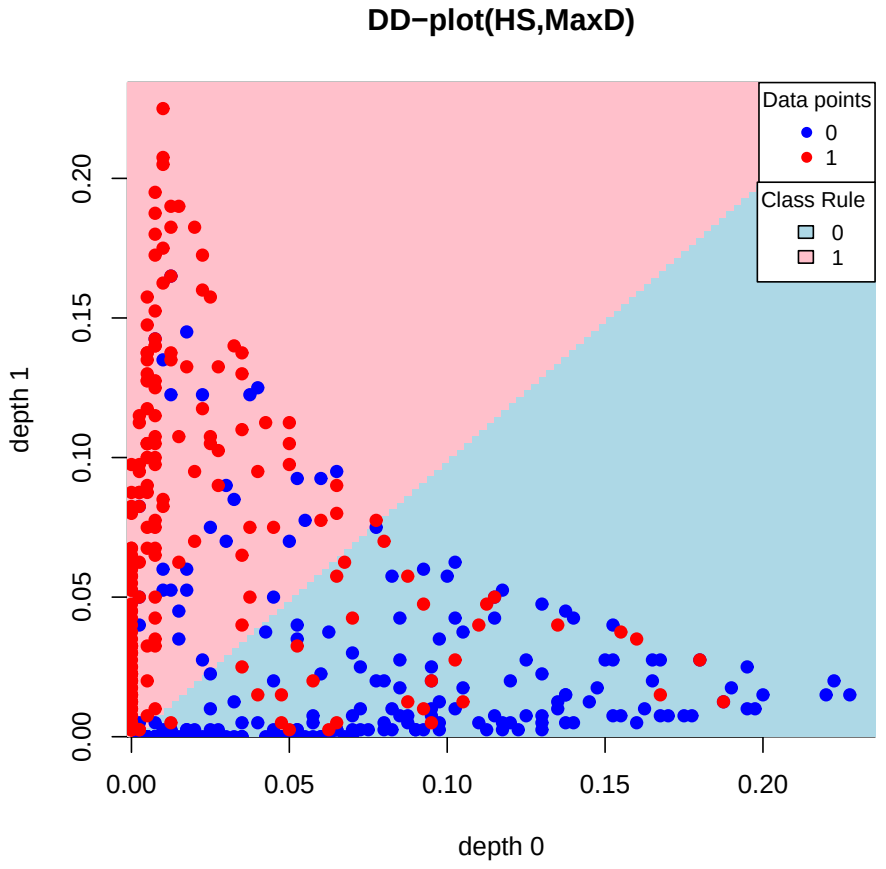
Prediction error: DD-HS=0.186, DD-SD=0.179





Prediction error: DD-HS=0.186, DD-SD=0.179, LDA=0.173, QDA=0.175, RF=0.21





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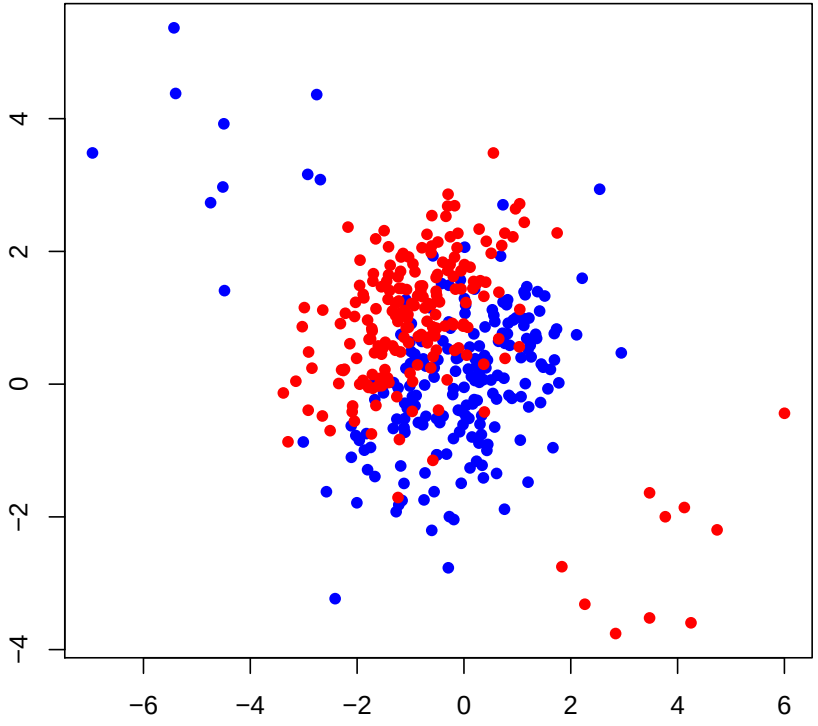
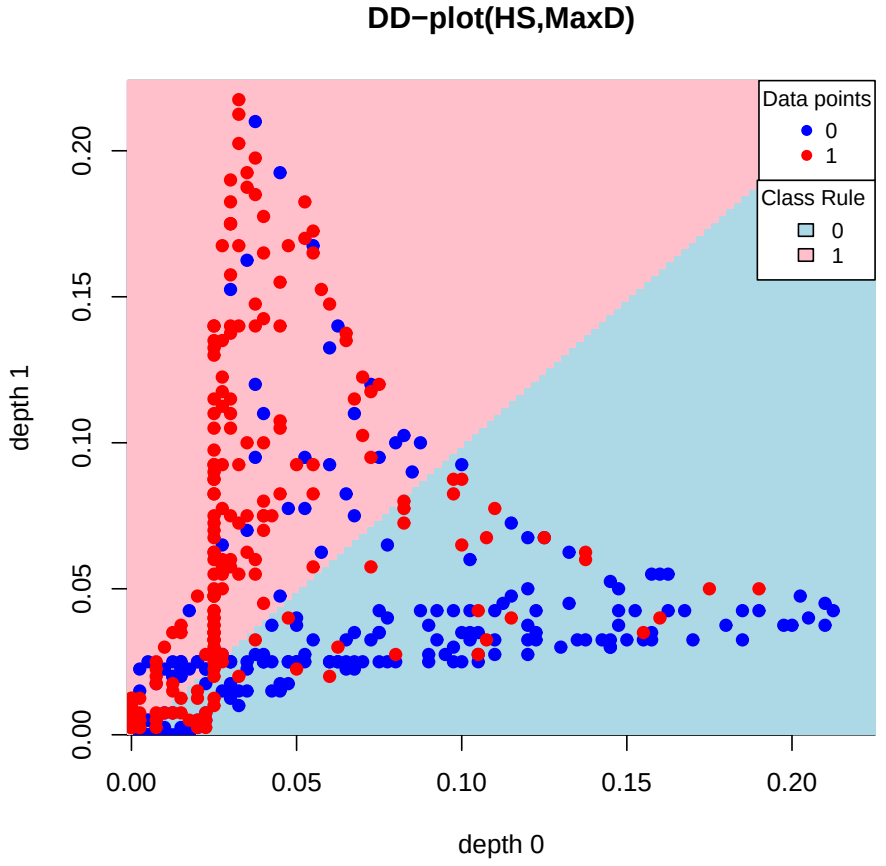
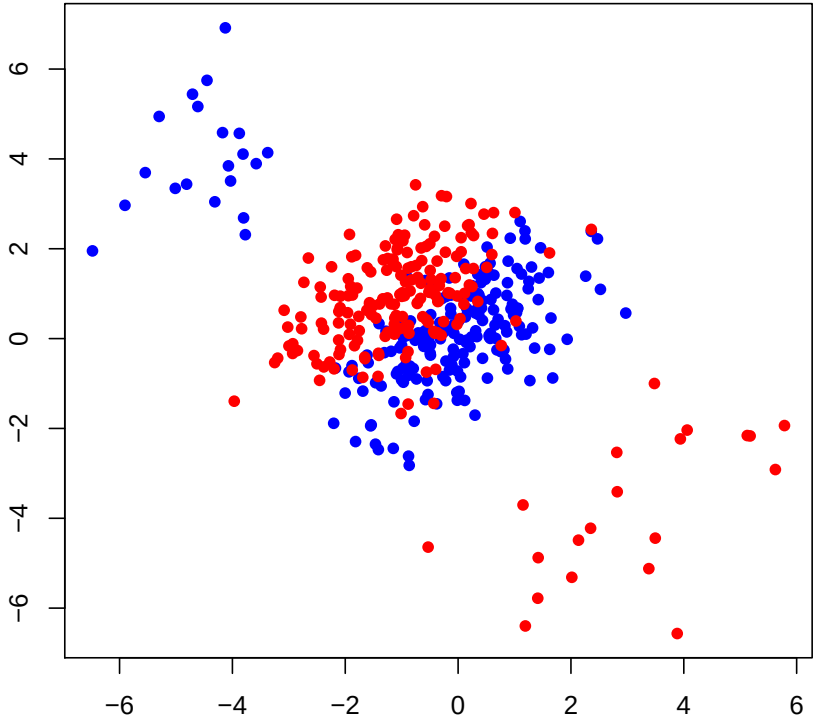
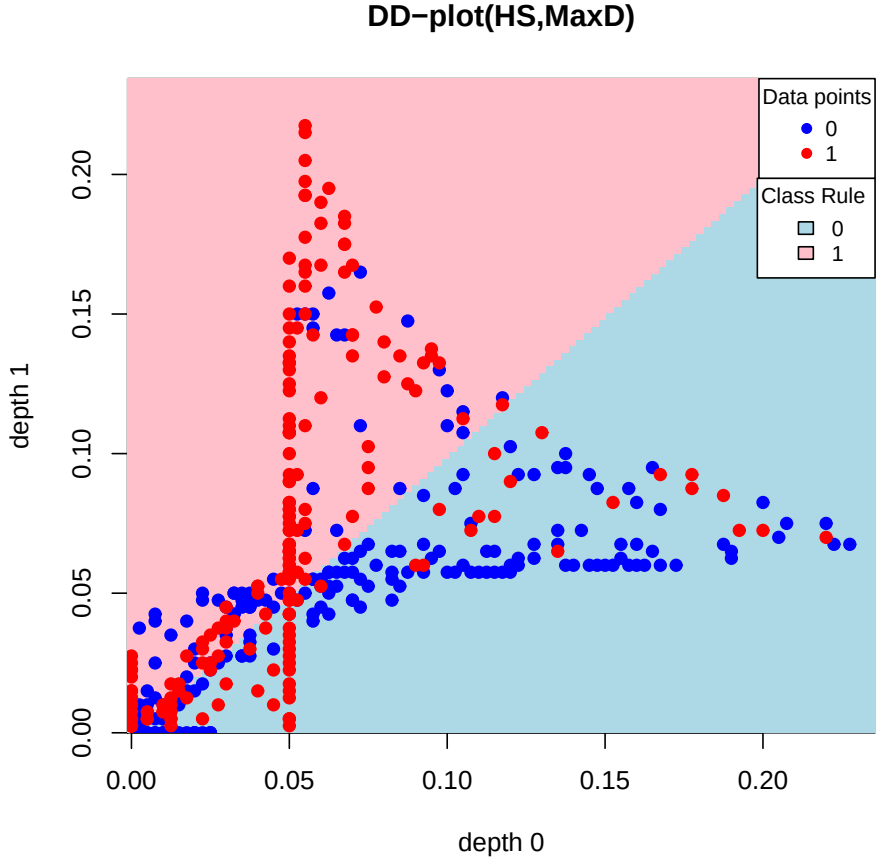
Prediction error (5% contamination):
DD-HS=0.306, DD-SD=0.297,
LDA=0.218, QDA=0.238, RF=0.17



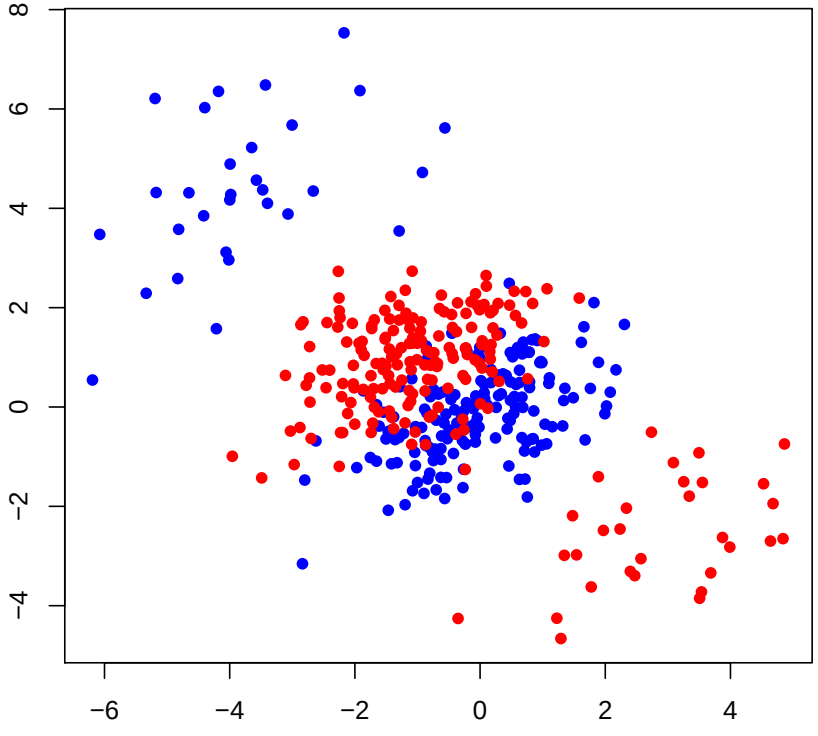


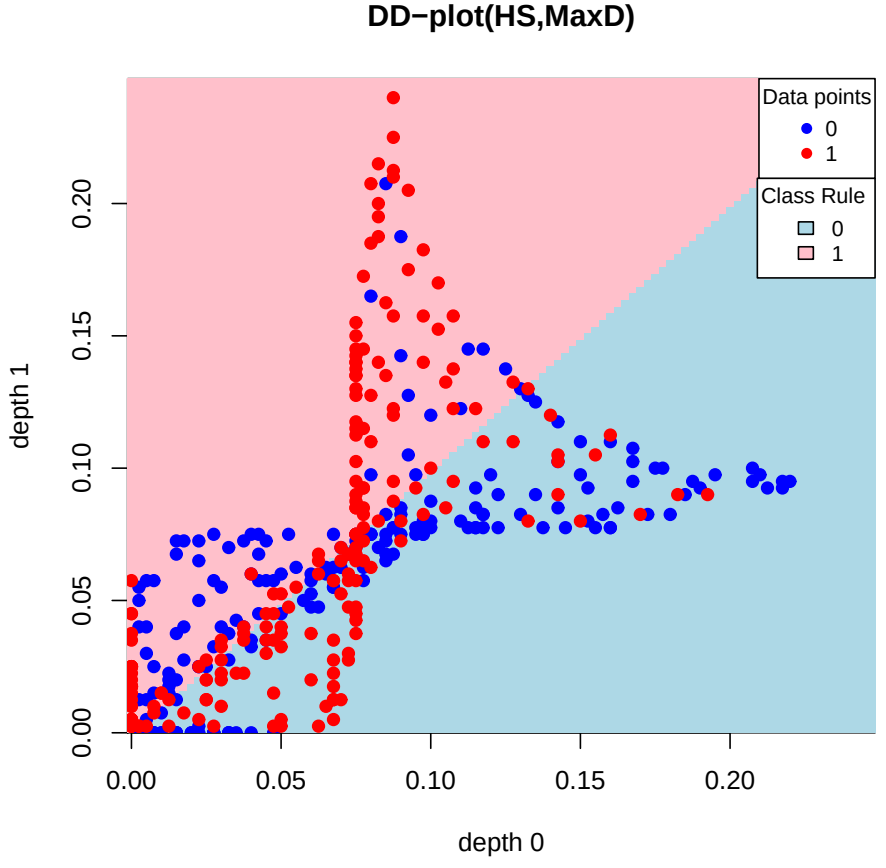
Prediction error (10% contamination):
DD-HS=0.4, DD-SD=0.35,
LDA=0.439, QDA=0.466, RF=0.176





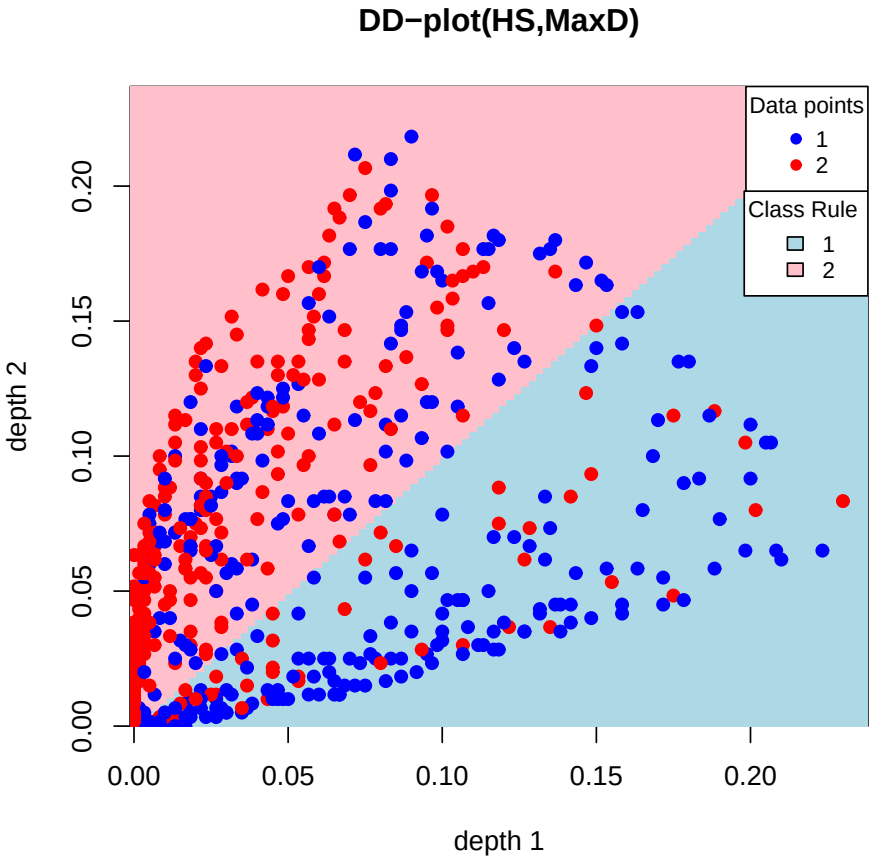
Prediction error (15% contamination):
DD-HS=0.511, DD-SD=0.424,
LDA=0.836, QDA=0.554, RF=0.184

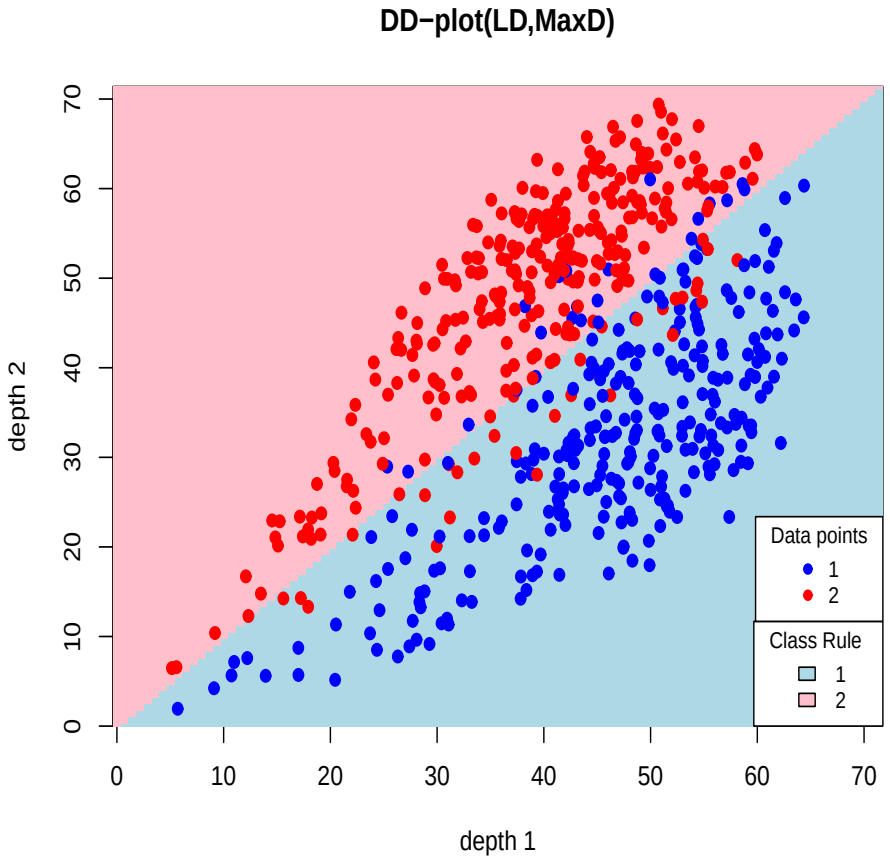




DD-plot for structure-borne noise data at two time points using maxDepth with

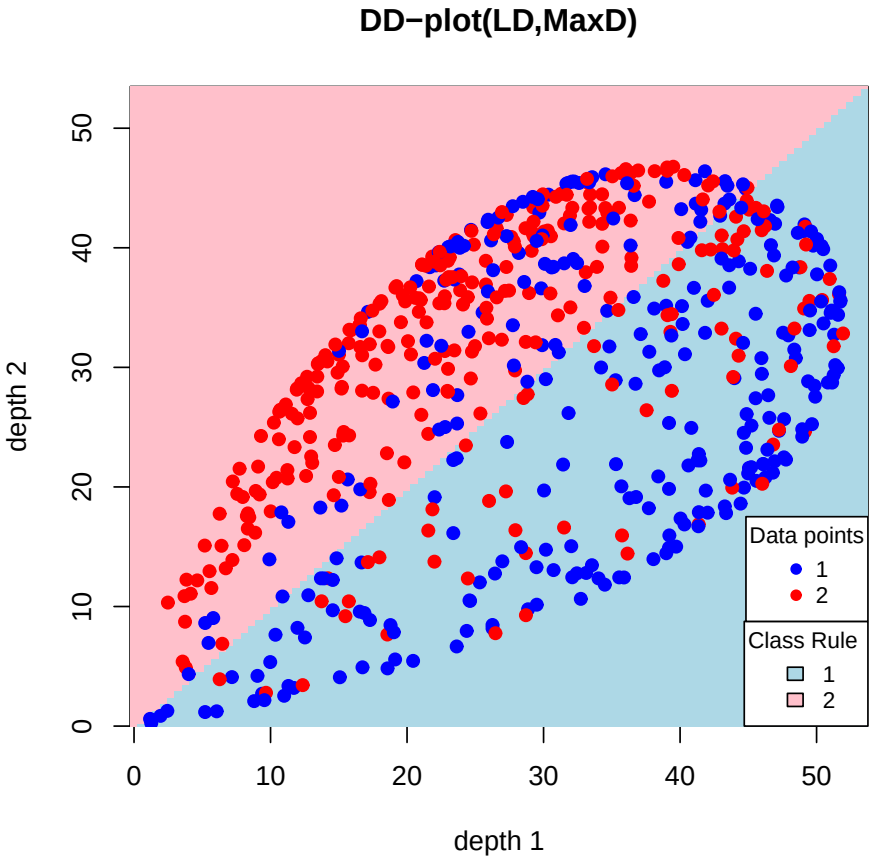
2 principal components or 6 principal components

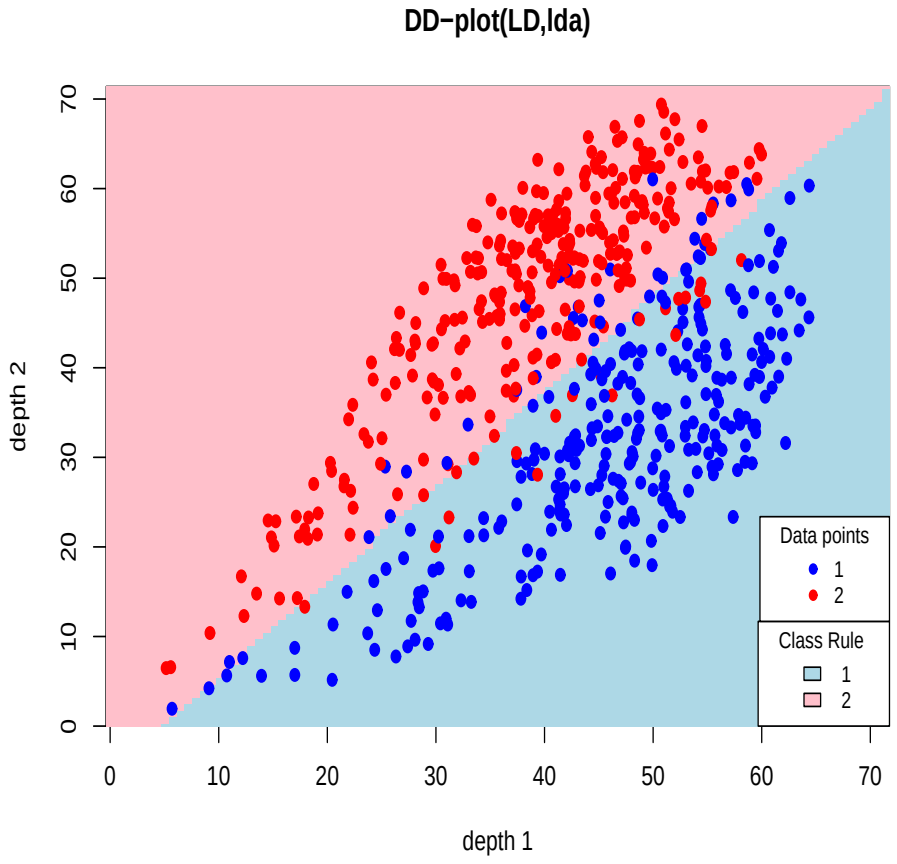




DD-plot for structure-borne noise data at two time points using LDA with

2 principal components or 6 principal components





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3. Data depth for classification

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- Classification can be done also for more than two groups. But then a DD-Plot is not possible.

Liu, R.Y., Parelius, J.M., und Singh, K. (1999). Multivariate analysis by data depth: descriptive statistics, graphics and inference, (with discussion and a rejoinder by Liu and Singh). *Annals of Statistics* **27**, 783-858.

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`http://cran.r-project.org/web/packages/depth/depth.pdf` (5.10.2015)

`https://cran.r-project.org/web/packages/fda.usc/fda.us` (5.10.2015)

`https://cran.r-project.org/web/packages/ddalpha/` (30.10.2015)